INTERACTION OF MODES IN MAGNETRON OSCILLATORS

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The understanding of mode problems in magnetrons has been obscured on account of misconceptions to an even greater extent, perhaps, than on account of absence of information on the subject. Some of these prevalent misconceptions are tacit, while others have broken into print. A sympton of failure to recognize fundamental problems is the widespread use of the expression "to mode" as a verb, which is applied to a magnetron any time that proper operation in the desired mode does not occur, without any careful thought as to what is actually taking place. Of all the material published on the subject, only that by Rieke and by Fletcher, both of the M. I. T. Radiation Laboratory, has been of substantial value with respect to the work reported here. It was these two authors who first reported on the distinction between the failure of a magnetron to start in the desired mode, _misfiring_, and mode changes after oscillation in one given mode started.

The results reported here are almost all qualitative. It has been the primary object of this research to find out the kind of thing which happens in magnetrons; obtaining numerical answers which could be applied to magnetrons in general has been considered beyond the scope of work intended here.

The author wishes to express his appreciation to his supervisor, Professor S. T. Martin, who suggested this topic for research, has encouraged it, and has been the source of many valuable suggestions. He also wishes to acknowledge the contribution made by others of the staff of the Research Laboratory of Electronics, including Professor W. P. Allis, who has been very helpful in assisting with theoretical work; Mr. L. B. Smullin, who has made some very important suggestions on experimental techniques; and Mr. W. E. Vivian who first suggested the application of van der Pol's non-linear oscillator theory to magnetrons. In addition, he wishes to thank Professor I. A. Getting and Professor E. A. Guillemin, who have taken an interest in this work and have made valuable suggestions, in spite of the fact that it is outside of their primary fields of interest. Mr. W. C. Brown and Mr. E. N. Kather of the Raytheon Manufacturing Company, have provided considerable information on the construction of magnetrons, which has been of great value in interpreting the observed results.

CHAPTER I

INTRODUCTION

1. The Problem

One of the most perplexing problems in magnetron work has been the "moding" problem. This problem arises because of the multiplicity of resonances in the resonant anode structure, and the fact that it is possible for the electron stream to support oscillation in any one of several modes of resonance. Also associated with the "moding" problem is the fact that under certain circumstances, the oscillating mode may fail altogether, and this event may or may not be followed by the starting of another mode of oscillation. Causes for mode failure are often obscure, especially when such failure is accompanied by the starting of oscillation in another mode.

In order to achieve proper operation of a magnetron, it is necessary to establish stable large-amplitude oscillation in one and only one of these modes. Some of the most widespread applications of magnetrons require microsecond-pulse operation. In magnetrons designed for such operation, it is necessary to establish oscillation in the desired mode positively and quickly, and to maintain it stably for the duration of the pulse. In magnetrons designed for c-w operation, quick starting is usually not a requirement, but stability is as important as in pulse magnetrons. Thus, the problems to be discussed here are of two kinds. First, the mode selection problem involves the establishing of oscillation in the desired mode positively and quickly, in pulse magnetrons. Second, the mode stability problem involves keeping the established oscillation stable, in either pulse or c-w magnetrons. Either of these two problems may be concerned with the avoidance of oscillation in unwanted modes.

The significance of the mode stability problem is emphasized by the fact that one of the limits on high power in magnetrons is the maximum power for which the desired mode of oscillation is stable.

2. Nature of Magnetron Modes

The magnetrons under discussion here consist of a cylindrical cathode, a cylindrical electron-interaction space between cathode and anode, in which electrons must supply energy to the r-f field, and a resonant anode structure which is usually part of the outer shell of the magnetron, and which is coupled to the external r-f circuit. The usual type of anode structure consists of an even number of cavities equally spaced around the periphery of the anode, as shown in Fig. 1. In this figure, all of the cavities are alike. In considering the r-f fields in the

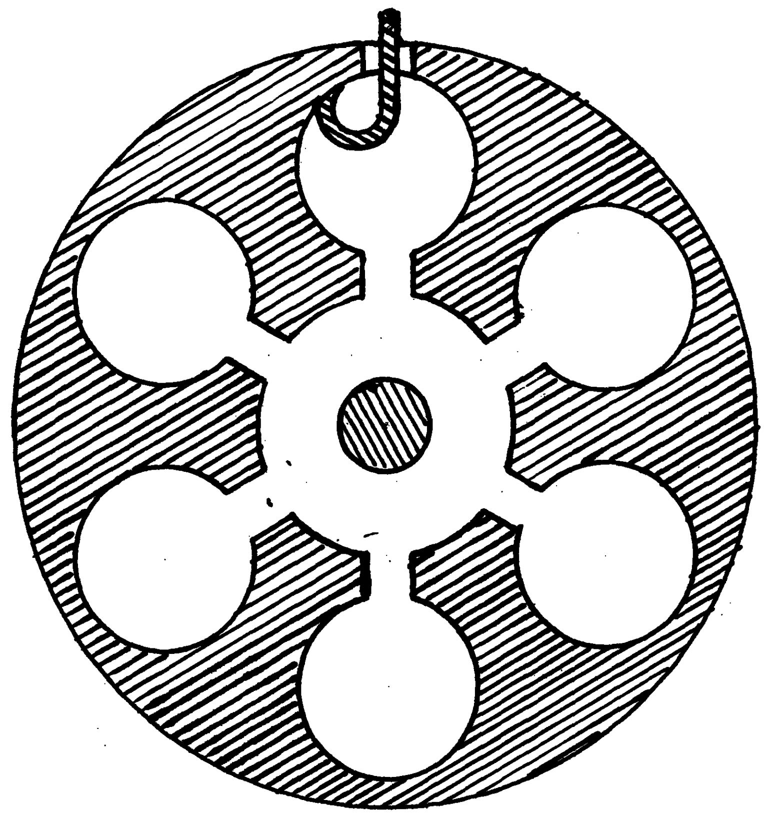


Figure 1. Cross-section of hole-and slot magnetron, with cathode in place, and with loop output.

resonant system, it is assumed that each of the N identical cavities has a uniform electric field across its mouth at any instant, and that there is a resultant travelling wave in the θ-direction, to which the electrons may be coupled and supply energy. (The system is described by the cylindrical coordinates r, θ, and z.) The field across the mouth of the m^{th} cavity is considered to be described by the expression: $E \exp(\frac{j2\pi nm}{N})$. Here, the mode number, n, is an integer, for it is necessary that $\exp(\frac{j2\pi nm}{N}) = \exp(\frac{j2\pi n(m+N)}{N})$ for all n, since the $(m+N)^{th}$ cavity is the same cavity as the m^{th}. A typical r-f electric field configuration is shown in Fig. 2.

If the N cavities are not identical, as in the rising-sun type of anode structure, the picture remains the same in principle. The rising-sun anode structure consists of alternate shallow and deep cavities, as shown in Fig. 3. In this case, the r-f electric field intensity across each

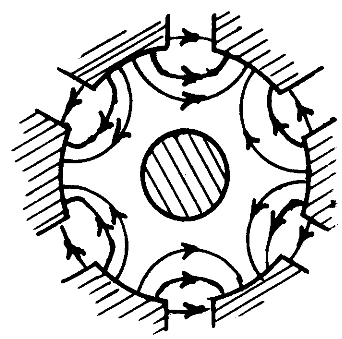


Figure 2. R-f electric field configuration in the interaction space of a magnetron.

slot may be described by: $A \exp(\frac{j2\pi nm}{N})$, for the deeper cavities, letting m be even; it may then be described by: $\alpha \exp(\frac{j2\pi nm}{N})$, for the shallower cavities, letting m be odd. If the depth of the shallower cavities approaches that of the deeper cavities, the value of α approaches that of A.

There are other types of magnetron anode structures in which the resonant structure is less intimately attached to the anode. An example is the interdigital magnetron. Here, the anode consists of an even number of finger-like anode segments, with alternate segments attached to continuous rings at opposite ends of the anode. An example of an interdigital anode is shown in Fig. 4.

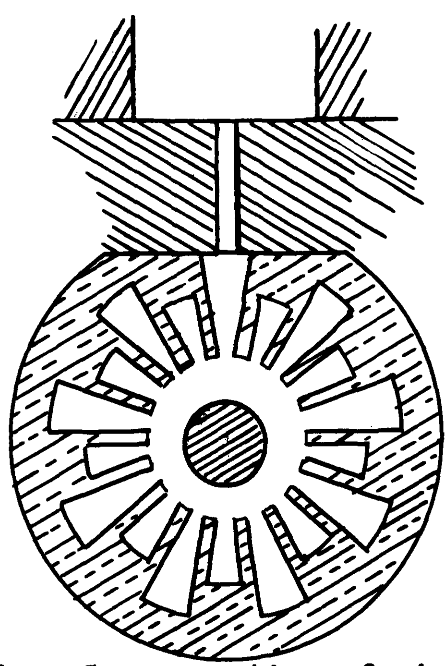
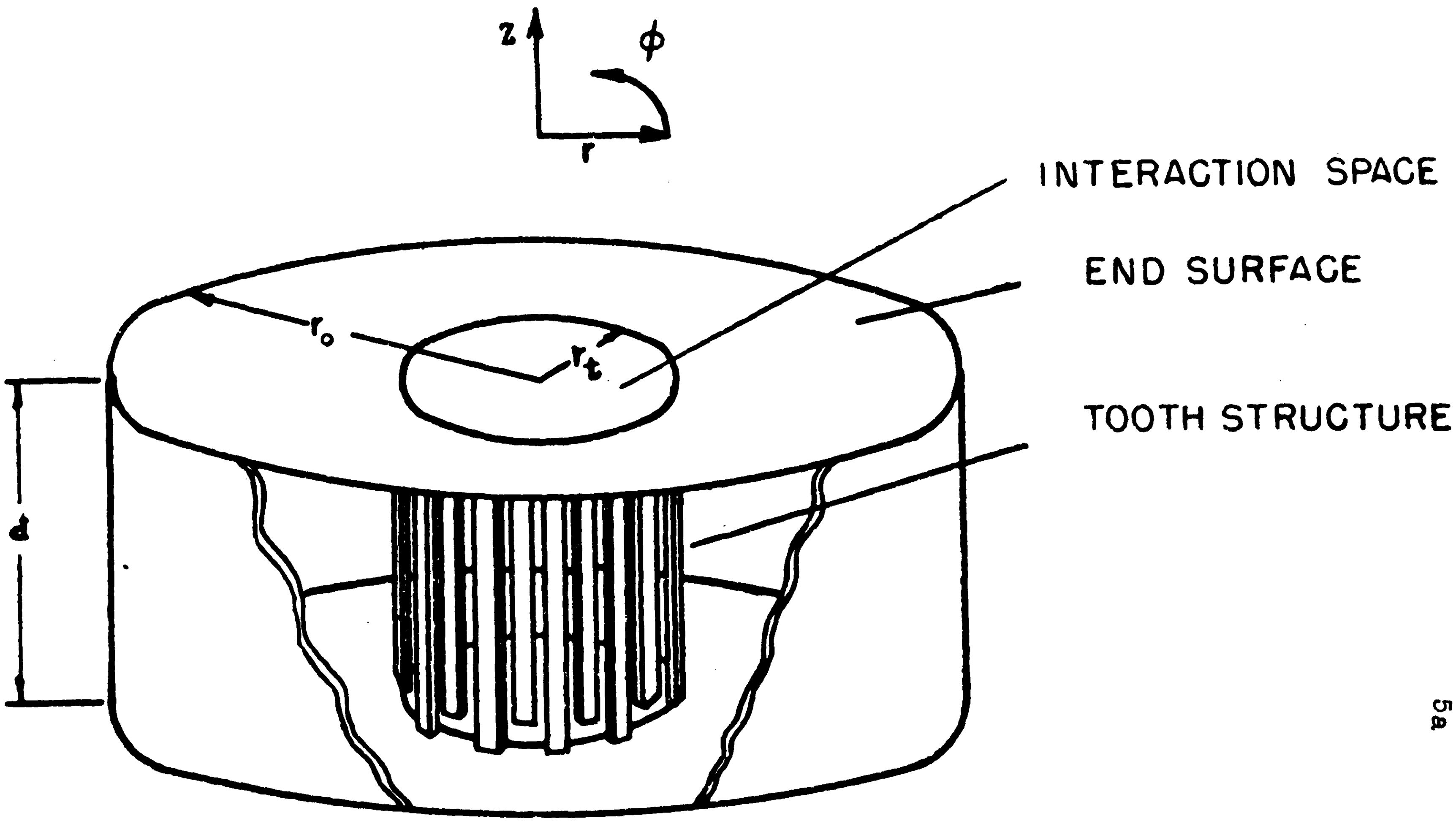


Figure 3. Cross-section of rising sun magnetron, with waveguide output.

Figure 4. Interdigital magnetron with "pill-box" resonator.
(From reference no. 29, p. 1259.)



Here, the rings on opposite ends of the anode have been extended to make up part of the structure of a pill-box-shaped resonator. The resonant structure may also be entirely outside the evacuated envelope, and in such a case may consist of lumped circuit elements or of a short-circuited transmission line.

The discussion which follows will apply primarily to multi-cavity magnetrons, except where it is stated otherwise. However, the electron motion studies and the equivalent circuit concepts should be applicable to either multi-cavity or interdigital magnetrons.

The general form of the solution of the wave equation for the electric field in the interaction space, assumed to be an infinitely long cylinder, is: $E \exp(jp\theta) Z_p(kr)$, where Z_p indicates a linear combination of Bessel and Neumann function of the p^{th} order, and k is the propagation constant such that $k = \frac{2\pi}{\lambda}$, where λ is the free-space wavelength at the frequency considered. To match the solution to the boundary conditions at the inside periphery of the interaction space, Hartree[1] resolved the r-f electric field into Fourier components in space, and these components are called Hartree harmonics. In an anode structure where all cavities are alike, the values of p for which r-f field components are actually present in the n^{th} mode are given by $p = n + \nu N$, where $\nu = 0, \pm 1, \pm 2$, etc. If

(1) Reference No.16. (See Bibliography.)

the cavities alternate between two sizes, as in the rising sun anode structure, $p = n + \frac{N}{2}$.

The field configurations described above lead to N possible modes. The actual pattern around the anode is described by:

$$\sum_{\nu=-\infty}^{\infty} E_p \exp j(\omega t - \frac{2\pi}{p}\theta) + \sum_{\nu=-\infty}^{\infty} E_p' \exp j(\omega t + \frac{2\pi}{p}\theta)$$

Thus there are two sets of waves, travelling in opposite directions. Only one such configuration, and therefore only one mode, can exist for $n = \frac{N}{2}$ and for $n = 0$. However, the solutions of the wave equation for other values of n in a perfectly symmetrical anode structure are not unique because the phase and amplitude relationships between the two sets of travelling waves are not determined. The presence of a coupling to the external circuit at one cavity removes the degeneracy of the solution,(1) and electric fields of the form:

$$E \cos \frac{2\pi n m}{N}, \text{ and}$$

$$E \sin \frac{2\pi n m}{N}$$

where $m = 0$ at the output cavity, are solutions if the

(1) Reference No.3, p.215.

loading is small. Therefore, there are two solutions for each integral value of n between n = 0 and n = $\frac{N}{2}$. If n is greater than $\frac{N}{2}$, say $\frac{N}{2}$ + 1, it is easy to show that a travelling wave in the opposite direction, with n = $\frac{N}{2}$ - 1, has the same field configuration as the original travelling wave, and therefore n = $\frac{N}{2}$ + 1 does not represent an additional mode.

It is significant that a resonance characterized by an r-f electric field of the form:

$$E \sin \frac{2\pi nm}{N}$$

where m = 0 at the output cavity, is very lightly loaded. This condition follows from the fact that there is a very weak r-f electric field at the mouth of the output cavity, and therefore very little energy is carried out through the output circuit.

Oscillation in a given mode takes place when the rotating electron stream couples to a travelling wave corresponding to one of the Hartree harmonics of one of the resonant modes described. Under the proper conditions, a regenerative action takes place in which the r-f electric field tends to bunch electrons in synchronism with itself, and in such phase that the electrons give up energy to it. Operating modes are designated by γ/n/N, where γ represents the particular Hartree harmonic to which the electrons are coupled, and n and N have already been defined above. It is convenient to visualize the electron configuration as being in the form of γ spokes, as shown in Fig. 5. The π-mode in an eight-cavity magnetron, shown in the figure, is designated 4/4/8.

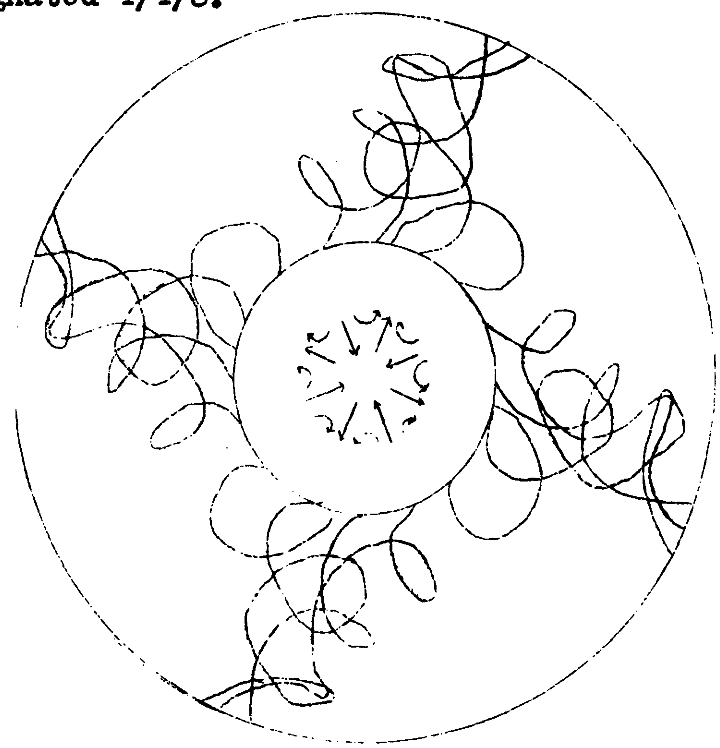


Figure 5. Electron paths in oscillating magnetron, showing "spokes Coordinates are rotating in synchronism with r-f wave. Arrows in center indicate direction of r-f electric field. (From reference no. 4.)

3. Mode Selection

If an electron is to fall into synchronism with a rotating r-f wave in a cylindrical magnetron, it is necessary that its velocity in absence of the r-f field be somewhere near the velocity of the rotating wave. This is particularly true during the build-up transient, when the r-f electric field which is available to act on the electron, and thus maintain synchronism, is small. According to Slater,[1] in order that an electron in a circular orbit at radius r_o should have a velocity v, the following conditions must be met:

$$E = v\left(B - \frac{m}{e}\frac{v}{r_o}\right)$$

(E = electric field intensity, B = magnetic flux density, m = mass, and e = charge of the electron; units are rationalized m.k.s.) From this he estimates the d-c potential difference between anode and cathode for synchronism between electrons and r-f travelling waves. Hartree[2] has developed an expression for the minimum d-c anode voltage for an electron to reach the anode in the presence of an infinitesimal r-f rotating wave. Hartree's expression for voltage, like Slater's, is a function of the velocity of the r-f rotating wave, of anode and cathode diameters, and of magnetic flux density. Hartree's criterion provides better agreement with observed magnetron operation than Slater's, although they are qualitatively similar in most respects.

The above discussion has been included for the purpose of showing that the most important factors in determining

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(1) Reference No. 4, p.107.
(2) Reference No. 16.

mode selection in a given magnetron are the applied voltage and the magnetic field. Thus, in many cases, for a given magnetic field, the mode of oscillation which is selected is primarily a function of applied d-c voltage. In other cases there may be two modes possible for a given applied voltage, and mode selection is less certain. The latter condition has been discussed in detail by Rieke,(1) and will be discussed further in later chapters.

4. Mode Stability

It has been mentioned above that the magnetron becomes an oscillator when the r-f field bunches the electrons in such a manner that the electrons will in turn give up energy to the r-f field. (The bunching mechanism will be discussed in detail in a subsequent chapter.) Therefore it is necessary that to some extent, the r-f field will keep electrons in synchronism with itself, in spite of a tendency to go faster or slower. If the applied voltage is too high, the electron stream will tend to go too fast, and it may no longer be possible for the r-f field to keep them in synchronism. Then, oscillation in that mode will collapse, and oscillation may start in another mode, or it may be that no oscillation will start at all.

Another phenomenon occasionally met with takes place when, with oscillation in one mode taking place, oscillation

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(1) Reference No.1, Chapter 8 (by F.F.Rieke).

in another mode builds up, suppressing the original one. It will be shown in later chapters that the large-amplitude oscillation in the original mode tends to discourage such an event, but does not necessarily prevent it altogether.

A change from one mode to another during a pulse (or after oscillation in one mode has become established, in a c-w magnetron) is referred to as a *mode shift*; in particular, if such a change takes place so quickly that no transition range is observed, the phenomenon is called a *mode jump*.

CHAPTER II

HISTORY OF MODE PROBLEMS

1. Development of Anode Structures

The first cavity magnetron of the type now in widespread use is generally credited to Boot and Randall, at the University of Birmingham, England, in 1940.[1] A cross section of a magnetron anode similar to the Boot and Randall magnetron is shown in Fig. 1 (Chapter I). In the cavity magnetron, the resonant circuit normally external to a split-anode magnetron was replaced by a series of resonant cavities, which were integral parts of the anode block.[2]

This type of anode structure must inevitably have several modes of resonance. If coupling between cavities is neglected, the resonant frequency of the system is the resonant frequency of each of the cavities. Coupling between cavities tends to separate the frequencies of the various modes. In six-cavity anodes, frequency separation between the desired π-mode resonance and the nearest undesired ($n = 2$) resonance of about 3% has been achieved (700A-D magnetrons).[3] This separates the frequencies by several times the band width of the $n = 3$ resonance, and thus the excitation of the $n = 2$ resonance

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(1) Reference No.6.
(2) Reference No.3, pp.181-182, 209-214.
(3) Ibid., p.274.

by oscillation in the π-mode is small. (Coupling between these two modes takes place principally on account of the disturbance of the r-f patterns due to the output circuit.)

In magnetron anode structures with eight or more resonators, coupling between resonators was less, and therefore, frequency separation between the π-mode and others was less. In some of these eight-segment anodes, separation between the π-mode ($n = 4$) and the $n = 3$ resonance was comparable with the band width of the loaded π-mode (e.g., 706A-C, and 714A magnetrons)[1] and therefore the π-mode r-f field could be contaminated by the presence of field components corresponding to $n = 3$. In such magnetrons, efficiency was poor, and attempts to operate at high power levels were often accompanied by mode jumps. Outward evidence of mode jumps included a small but appreciable change in operating frequency, and considerable changes in the input current and voltage values. Such a mode change could be made very quickly because of the fact that stored energy in the $n = 3$ mode was already present, and therefore the usual time for the build-up of a mode of oscillation would be very much reduced.

In an effort to prevent oscillation in any mode other than the π-mode, Sayers[2] modified an anode structure of

(1) Reference No.3, p.299.
(2) Reference No.13.

the type discussed above by connecting alternate anode segments with wire "mode-locking" straps. He expected that, since only in the π-mode are alternate segments of the anode at the same instantaneous potentials, oscillation in other modes would be virtually impossible.

The results of strapping were unexpectedly good. Not only was the mode jump, of the kind described above, eliminated, but efficiency was radically improved. Electronic efficiencies of 50% or more were now possible, instead of 20% or less. The cause for such improvement seems to have been the removal of the undesired r-f field components mentioned above.

The actual effect produced by the straps was not the complete removal of the unwanted modes, because the inductance of the straps is appreciable. Strapping did, however, have the effect of producing wide separation between the π-mode frequency and the frequencies of other modes. In practice, the results were good enough so that "moding" difficulties were no longer present in many cases.

In trying to build magnetron oscillators for the 3-cm, and more especially the 1.25-cm wavelength bands, the small size of straps led to difficulties in manufacture, and to low circuit efficiency. In the first experiments intended to separate modes in frequency without straps, dimensions in certain particular cavities were altered in an effort to change the resonant frequencies of some modes more than others, with the expectation of improving the operation of non-π-modes.[1] These experiments did not lead to satisfactory magnetrons. However, by representing magnetron cavities approximately by means of equivalent circuits, it was found that the rising-sun anode structure, consisting of alternate deep and shallow cavities, led to adequate separation of π-mode frequency from frequencies of other resonances. (Cf. Fig.3, Chapter I.)

2. Mode Selection

In Chapter I it was brought out that magnetron operation of the kind discussed here requires that the average rotational velocity of an electron in absence of the r-f rotating wave be comparable with the velocity of the wave itself. Slater's estimate of magnetron operating voltage and Hartree's threshold voltage were mentioned there. In Fig. 6, the Hartree voltages for 3 modes in a typical eight-resonator magnetron are shown as functions of magnetic field. Also shown is the d-c cut-off curve, at which it is possible for electrons leaving the cathode with zero velocity to reach the anode in the presence of constant fields.[1]

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(1) Reference No.3, p.177.

Hartree[1] was apparently the first to point out the possible rotating wave components associated with any given mode in a multi-segment resonant anode structure, and that it is possible for the electron stream to become coupled, under different conditions, to different components corresponding to the same mode of resonance.

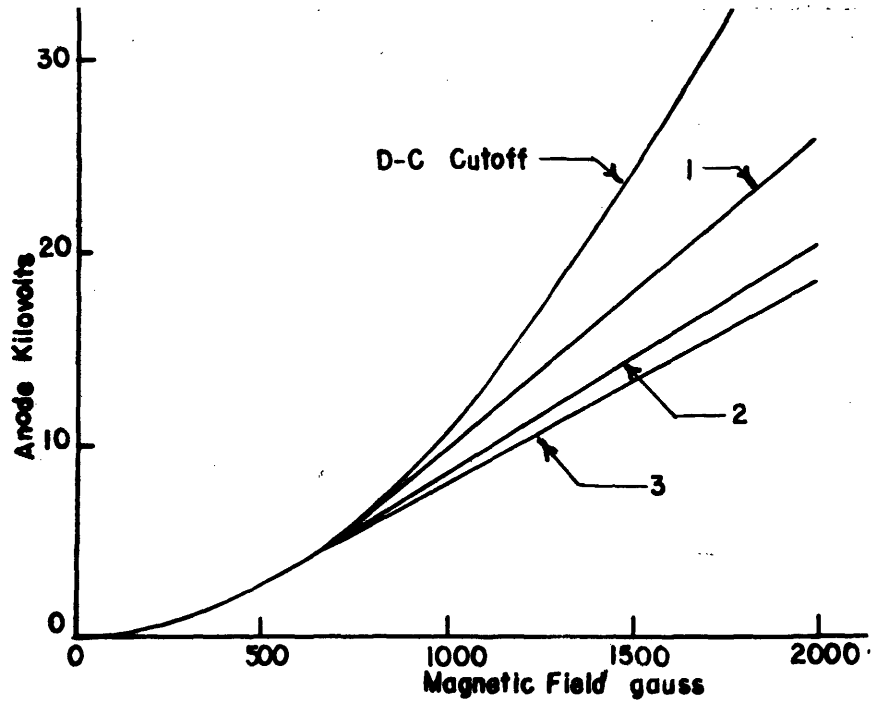

Figure 6. Hartree diagram for 718 EY magnetron.
Threshold voltages: (1) 3/3/8 mode
(2) 4/4/8 mode (π-mode)
(3) 5/3/8 mode

Bunemann[2] uses another approach altogether to establish a necessary condition for oscillation. He assumed first the existence of the <u>Brillouin steady state</u>, which represents a solution to the electron motion problem. In such a solution, the radial current is zero, and the

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(1) Reference No.16; also reference No.1, p.340.
(2) Reference No.1, Chapter 6 (by L.R.Walker), p.253.

electrons are all moving in concentric circles. This is also referred to as a single-stream solution of the electron-motion problem. (It has been shown by Twiss[1] that a single-stream solution is impossible in the presence of a Maxwellian distribution of emission velocities, and that a double-stream solution is always possible under such circumstances. The single-stream solution may be approached as a limit as emission velocities approach zero. The question of how the electrons in a magnetron could ever fall into the single-stream solution was also raised.)

Under certain conditions of voltage and magnetic field, a perturbation of the single-stream steady state corresponding to an r-f rotating-wave may cause the space charge to become unstable, and spokes (cf. Fig. 5) will build up, thus initiating oscillation. When such a voltage, corresponding to regeneration in a particular mode, is reached, that voltage is called the instability voltage for that mode.

Instability voltages have been computed,[2] and it is found that for values of γ of four or less, the instability voltages are comparable with threshold voltages and represent plausible standards for minimum starting voltages in terms of observed starting voltages. For larger values of γ , the instability voltages become further re-

(1) Reference No. 31.
(2) Reference No. 22.

moved from both the threshold voltages and observed starting voltages. Thus, it is necessary to conclude that the instability voltage criterion is, in general, not applicable to actual magnetrons. On the other hand, threshold voltages are never in very great disagreement with observed starting voltages.

Fletcher and Rieke(1,2) have pointed out the importance of pulse modulator characteristics, especially the rate of rise of the voltage pulse applied to the magnetron, and the pulser's output impedance. They have shown how a pulse may rise so rapidly that it passes through the range in which the desired mode can start before oscillation in that mode can build up appreciably, and into a region where an undesired mode, or no mode at all, can start. If oscillation in the desired mode could have built up more quickly, or if the rise in voltage had been slower, oscillation would have caused d-c current to flow, and the flow of current would have held the voltage down to a value where oscillation in the desired mode could have persisted. On the other hand, it is pointed out that if the open-circuit voltage of the pulser falls within the range in which the desired mode can start, or if the rate of rise of the pulse can be reduced, it should be possible to eliminate misfiring (that is, failure to start in the desired mode).

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(1) Reference No. 14.
(2) Reference No. 1, Chapter 8 (by F.F.Rieke).

The problem of competition between modes, which occurs when the anode voltage of the magnetron is rapidly raised from zero to a value at which oscillation in either of two modes could be supported, is discussed by Rieke.[1] The process of one mode gaining ascendancy over the other and eventually suppressing it is described from a non-linear circuit point of view. In particular, he points out that when the amplitudes of oscillation in both modes are large enough that non-linear effects are important, it is necessary that the amplitude of one mode affect the rate of build-up of the second more than it affects its own rate of build-up. Such a condition is necessary in order that mode selection be definite on any particular pulse. This condition does not preclude uncertainty of selection as between successive pulses of the same amplitude.

Here, Rieke has described the assumption that the amplitude of one mode affects the rate of build-up of the other mode more than its own rate of build-up, as being open to question. Nevertheless, it is evident that it must be valid for all, or at least nearly all, of the observed cases, because of the definiteness of mode selection on any particular pulse. In a later chapter, theoretical reasons tending to confirm Rieke's assumption will be advanced.

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(1) Reference No.1, Chapter 8 (by F.F.Rieke).

3. Mode Stability and Mode Changes

In the preceding section mode selection was discussed as a transient problem. In this section mode stability is to be considered as primarily a steady-state problem. The only kind of magnetron in which a strictly steady-state analysis is applicable is the c-w magnetron. In many stability problems in pulse magnetrons, the time required for appreciable changes in operating conditions is very long in terms of r-f cycles, and the steady-state analysis is entirely acceptable. In many other pulse magnetrons, a borderline condition between mode skip and mode shift is encountered. This borderline situation is considered in this section, although some of the fundamental principles mentioned in the preceding section apply.

It should be emphasized here that starting criteria may not necessarily be expected to apply for a given mode, once large-amplitude oscillation has been established in another mode. For example, the Hartree starting criterion, that is, the threshold voltage, is no longer as applicable as it was before oscillation started. This criterion specifies the anode potential at which an electron can just reach the anode in the presence of an infinitesimal r-f wave rotating with a given velocity, and in the presence of a given magnetic field, for any particular magnetron, assuming zero emission velocity. In the presence of large-amplitude oscillation in one mode, another r-f rotating wave of small amplitude can have little effect upon whether an electron reaches the anode. Furthermore, it is significant that the threshold voltage represents a necessary but not sufficient condition for oscillation.

Nevertheless, many of those who have worked with magnetrons have accepted starting criteria, which were intended to apply only to the given mode in the absence of others, as being equally applicable to the given mode in the presence of large-amplitude oscillation in another mode. Some of these ideas often seemed to be confirmed when magnetron design changes were made.

An example of such an idea, which became fairly widespread, at least tacitly, is that raising the starting voltage of the next higher-voltage mode above the π-mode will necessarily increase the maximum voltage at which the π-mode can be operated stably. There are two possible reasons for reaching this conclusion. First, the use of straps increased the upper limit of input current and input voltage for which a magnetron was stable in the π-mode. The straps also increased the resonant frequency for other modes, and therefore increased their respective starting voltages. Hence, the voltage of the π-mode could be raised further above the threshold value without reaching the starting voltage for another mode. Therefore, some were led to the condlusion that the fundamental improvement in stability resulted from the separation of threshold voltages. The presently accepted explanation for the principal cause of the improvement is that the separation of the resonant frequencies prevents contamination of the π-mode r-f pattern with components of other modes, as discussed previously in this chapter.[(1)] Another cause of confusion was the application of the expression "moding," both to improper starting and to instability of the desired mode. It is reasonable to expect that starting in the wrong mode becomes more difficult as its range of starting voltages becomes further removed from the starting range for the desired mode.

Another concept of mode stability based entirely on other possible modes has been advanced by Copley and Willshaw.[(2)] They first assume that both the threshold and the instability voltage criteria (see preceding section) must be met before oscillation takes place. After oscillation is established, the applied voltage may be increased indefinitely with corresponding increase in power, until the instability voltage for another mode is reached. At this point oscillation in the original mode ceases, but oscillation in the second mode will not start unless (or un-

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(1) Reference No.1, Chapter 4 (by L.R.Walker).
(2) Reference No.22; also Reference No.8, p.1000.

til) the threshold voltage for the second mode has been reached. Thus it is possible that there will be a range of anode voltages in which no oscillation will take place. Calculations showed that if the cathode diameter were increased, the values of threshold voltage decreased while values of instability voltage increased. Thus, the voltage range between the threshold voltage of the desired mode (in all cases under discussion by these authors, the π-mode) and the instability voltage of the next higher-voltage mode can be increased; since, according to the theory, the maximum input current is approximately proportional to this voltage range, to maximize this voltage is to maximize input power. If the increase in cathode diameter does not reduce efficiency too much, such a procedure should be expected to lead to greater output power.

The application of this mode-change criterion to actual magnetrons produced agreement with theory in some respects, at least qualitatively. The most significant bit of agreement was observed when the cathode diameters were increased. This change actually led to considerable increases in output, as well as input, power.

The latter theory of mode change is based on several assumptions which are very much open to question. Doubt as to the possible existence of a Brillouin steady-state has already been mentioned in the preceding section. Another doubtful point is whether the calculated instability voltages could have any significance when large-amplitude oscillation is already present. But a much more fundamental question has been raised by Dunsmuir.(1) The question is whether a lower-voltage mode, whose conditions for oscillation have already been met in terms of threshold and instability voltages, should necessarily give way to a higher-voltage mode as soon as the latter's conditions for oscillation are met.

Experimental evidence also refutes this mode change criterion, and any other general criterion based solely, or even primarily, on the unwanted modes. Some of the experimental work reported on in a subsequent chapter shows clearly a set of mode changes depending primarily on conditions in the originally oscillating mode, instead of the mode into which the change took place; in another test, π-mode oscillation is maintained stably as the applied voltage is raised past both threshold and instability voltages for at least one other mode, and it stops only when d-c cut-off is reached.

The magnetron improvements which were accomplished by application of the instability-voltage criterion can be explained otherwise. An increase in cathode radius leads to more stable operation because of the increased intensity

(1) Reference No. 9.

of the r-f electric field available near the cathode for bunching; the intensity of the latter field for the p^{th} order Hartree harmonic is proportional to r^p, where r, as mentioned before, is one of the coordinates of the cylindrical system. This effect will be discussed further later on in this chapter, and in subsequent chapters.

The maximum-current limitation in magnetrons had also been encountered at the Bell Telephone Laboratories.(1) Here, the problem was to design a magnetron in the 1220-1350 megacycle range (L-band) for high-power operation. The maximum current limitation in other L-band magnetrons had been recognized as being related to the rate of rise of the applied voltage pulse. It was further recognized that <u>the failure to operate in the π-mode was independent of the presence of other modes</u>.

This problem involves some of the aspects of both mode <u>skip</u>, or misfiring, (discussed in the preceding section) and the kind of stability problem met with in c-w magnetrons and in those with slowly rising pulses. A quantity, critical current (I_c) was defined as the input current at which there was first observed a narrowing of the current pulse at the leading edge. It was found that as the rate of voltage rise became less, I_c increased. A maximum value was approached, which could not be exceeded by any further decrease in the rate of voltage rise.

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(1) Reference No.18; also reference No.1, p.378.

A systematic experimental study was made to determine the effect of the variation of design parameters upon I_c. As in the magnetrons reported on by Copley and Willshaw (see above), it was found that stability of the π-mode was increased by increasing the ratio of cathode radius to anode radius; it was also found that lighter loading increased I_c. Unfortunately, each of these changes which might be made to increase I_c also would decrease electronic efficiency. The idea of building a magnetron which was very much dependent on a slow rise of applied voltage was rejected, because this would limit the versatility of the magnetron more than was considered desirable.

As a result of the above considerations, a magnetron was designed with a much larger ratio of cathode radius to anode radius than previous eight-segment magnetrons. The individual resonators were slot-shaped, rather than being of the hole-and-slot type; this feature reduced the total energy storage for a given loading of the electron stream, and thus permitted more rapid r-f build-up.

It should be taken into account that problems arising from high rate of voltage rise as compared with the rate of build-up are more acute in lower frequency magnetrons, such as L-band, as compared with S-band and higher frequencies. In magnetrons which are equivalent otherwise, but have different frequencies, the build-up rates <u>per cycle</u> should be the same.

An explanation for this kind of combined misfiring and stability problem apparently depends on the fact that the building up of oscillation causes current to flow from the pulser, and thus reduce the applied voltage, and also that greater r-f amplitude increases the voltage range for which electrons can be kept in synchronism with the r-f travelling wave. Therefore, the faster that r-f build-up can take place in relation to the rate of rise of the applied voltage, the greater the chance of stability. Thus, there is a continuous transition from the mode-skip transient problem to the steady-state stability problem.

Some experiments in magnetron design were made at the M.I.T. Radiation Laboratory for the purpose of making a mode <u>less</u> stable.[1] The test magnetrons all had K-7 (2J32) anodes, with anode diameter of 13.6 mm, and cathode diameters of 4, 5, 6, and 8 mm were used. The problem here was to discourage oscillation in the 5/3/8 mode which has a slightly lower starting voltage than the π-mode (4/4/8). Most efficient operation was found using the 4-mm cathode, but the cathode melted at moderate powers. Operation with 5-mm cathodes (for which the tube was originally designed) was satisfactory, while operation with 6- or 8-mm cathodes led to large areas on the performance chart where the unwanted mode interfered with operation in the π-mode.

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(1) Reference No.15.

The failure of a mode of oscillation as a result of the failure of the bunching mechanism to maintain synchronism between rotating electrons and the r-f field was at least implied by Slater in 1941.[1] It has also been discussed by this writer[2] and has been further investigated by the General Electric Research Laboratories.[3] The failure of oscillation is interpreted as being due to the tendency of a high d-c electric field to make the electrons rotate at a rate higher than that of the r-f traveling wave. These phenomena will be discussed in detail in the following chapter.

An attempt has been made recently by Welch and others at the University of Michigan to establish more definite mode stability criteria, and an estimate has been made of the space-charge limited current in an oscillating c-w magnetron.[4] This does not necessarily lead to an adequate criterion for the maximum current associated with stable π-mode oscillation, because space-charge limited current is a function of anode voltage, and there is yet no adequate theory which leads to a maximum voltage for stability.

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(1) Reference No.4.
(2) Reference No.19.
(3) Reference No.20.
(4) Reference No.35.

CHAPTER III

THE MAGNETRON AS A FEED-BACK OSCILLATOR

1. Energy Conversion in Oscillators

In order to make it possible to describe properly the characteristics of any particular mode of oscillation, the magnetron will be considered briefly here in terms of its properties as a feed-back oscillator.

In electronic oscillators in general, there is an active electronic system, which converts d-c energy into a-c energy, and there is a passive system which is frequency-sensitive and which controls electron flow or electron motion in the active system in such a way that the active system can produce a net output great enough to supply energy for losses and load. In the cylindrical magnetron, electrons tend to travel in more or less circular paths; if these electrons can be caused to rotate in synchronism with an r-f travelling wave, and to be bunched in the proper phase to give up energy to the wave, then energy conversion in the electron stream of the magnetron can take place.

In usual magnetron operation, the initial bunching results from one mechanism, and bunching is maintained by another. These have been described in some detail by Fisk, Hagstrum, and Hartman,(1) and the fundamentals are reviewed briefly below.

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(1) Reference No.3, pp.189-196.

Upon leaving the cathode with zero velocity, in the presence of non-time-varying fields (radial electric, axial magnetic field), the electron would start toward the anode, but its path would be bent by the magnetic field and it would return toward the cathode, reaching the cathode surface with zero velocity. If there is, in addition, an r-f wave rotating in the same direction as the electron, then the electron may receive from or give up energy to the r-f field. If it absorbs energy, it will be driven into the cathode with a finite velocity, and will be taken out of the system. If it gives up energy to the system it will return to a point of zero radial velocity in the space between cathode and anode. Thus it has had work done on it by the d-c electric field, and has done work on the r-f field. It is now in a position to stay in the system, and under favorable conditions, it may make more such loops toward the anode, giving up energy to the r-f field each time. When such an electron finally reaches the anode, it will have converted ideally most of the energy put into it by the d-c field into r-f energy. The electrons emitted in such a phase as to extract energy from the r-f field will be referred to as <u>unfavorable</u>, and those emitted so as to add energy to the r-f field will be called <u>favorable</u> electrons.

In the reference mentioned above,[1] the <u>phase-focusing</u>

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(1) Reference No.3, pp.189-196.

mechanism is also described. Under normal conditions, this mechanism will apply principally to favorable electrons, with the others removed from the system as they are driven back into the cathode. When an electron leads the position in the r-f travelling wave at which the electron would give up the maximum amount of energy to the tangential component of the r-f field, it is also acted upon by the radial component of the r-f field. The latter component is directed in such a way that, in combination with the axial magnetic field, the electron is accelerated toward the position at which it would give up maximum energy to the r-f field. On the other hand, a lagging electron is subjected to the action of the opposite r-f radial field, and is likewise accelerated toward the position where it would give up maximum energy to the tangential r-f field.

2. Build-Up Process

Small-amplitude phenomena in magnetrons are much less well understood than are large-amplitude phenomena. For this reason a completely satisfactory explanation of the build-up process is not yet possible. An attempt will be made here to supply a description of r-f build-up that is plausible from a qualitative point of view.

In the presence of only one r-f rotating wave, and with zero emission velocity, the rejection of unfavorable electrons should be complete as soon as any coherent r-f wave is present. In the presence of a Maxwellian distribution of emission velocities, it is to be expected that the rejection process will be incomplete for small r-f amplitude, and will increase in effectiveness with increasing r-f amplitude. During early stages of build-up, synchronism of electrons with the r-f wave will not necessarily be maintained all the way to the anode. In the first place, the average velocity toward the anode is small, being proportional to the r-f amplitude; in the second place, the effect of the r-f phase-focusing field may be expected to be small, in general, as compared with whatever tendency the d-c field, in combination with the magnetic field, may have toward making the electrons rotate at a velocity other than in synchronism with the r-f wave. Nevertheless, if the velocity of electrons is even approximately equal to that of the r-f wave, the bunched electrons, starting in the right phase (the unfavorable electrons having been rejected), should stay in the right phase long enough to lead to a net contribution of energy by electrons to the r-f wave.

As r-f amplitude increases further, the phase-focusing mechanism becomes increasingly effective in maintaining synchronism of the electron stream. The amplitude of r-f voltage necessary for effective phase focusing depends on the difference between the rotational velocity of the r-f wave and that of the electrons in the absence of the r-f field.

The final stage occurs when both of these bunching mechanisms are complete, and the electron stream is, to a first approximation, a constant-current generator. Then the r-f voltage (V_{rf}) approaches a limit (V_{max}) as follows:

$$V_{rf} = V_{max}(1 - e^{-\alpha t})$$

Some r-f build-up measurements were made by Fletcher and Lee[1] at the M.I.T. Insulation Research Laboratory. These results are plotted in Figures 7 and 8. Fig. 8 shows that in the first stage, the build-up process is exponential, and not very sensitive to anode voltage, once the process has started. (The later starting when anode voltages are lower may be due to the greater delay in reaching threshold voltage on the anode when peak amplitude is less, or to less noise in the electron stream.) This stage in Fig. 8 may well be associated with the first stage mentioned above, when the electron rejection process is becoming more effective. Above 200 watts r-f power output, the rate of build-up is more dependent on anode voltage. This may reasonably be associated with the second stage above, when rate of build-up is increasingly

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(1) Reference No.32; also cf.reference No.1, pp.373-376.

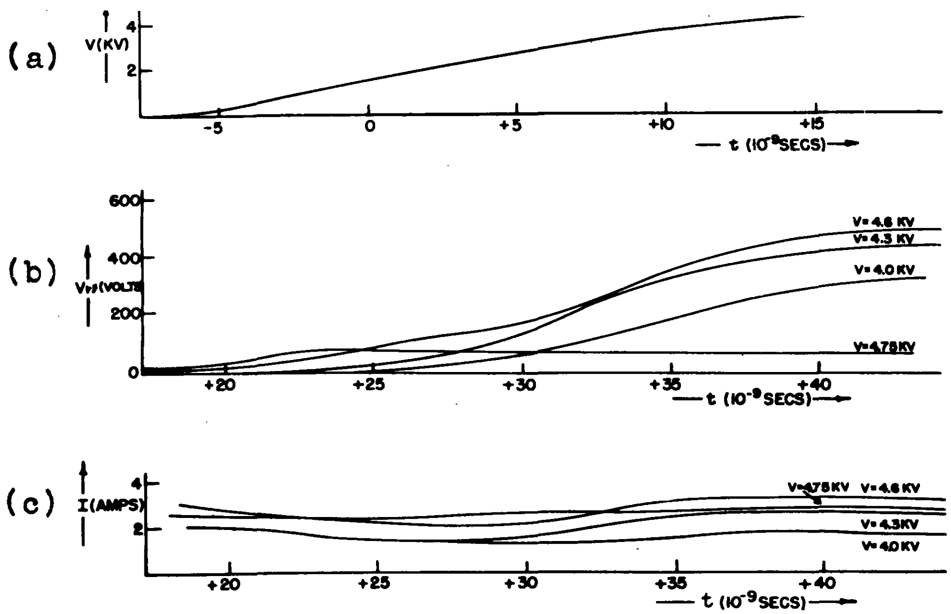

Figure 7. R-f build-up in magnetron. (a) Ap-voltage. (b) R-f envelopes for different applied voltages. (c) Anode currents for different applied voltages. (From reference no. 32.)

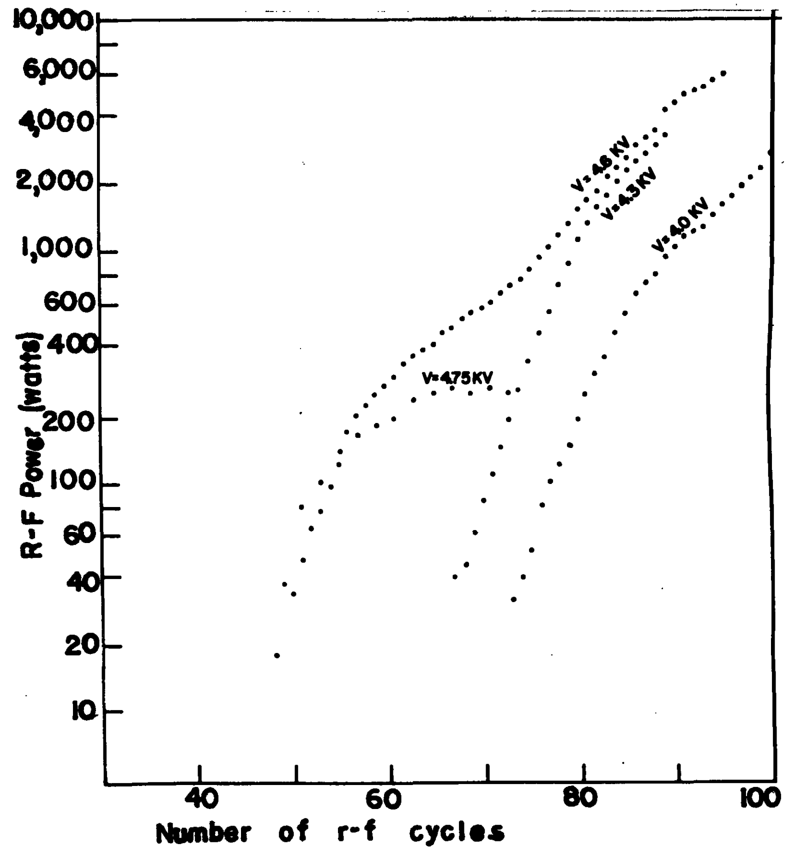

Figure 8. Logarithmic plots of the r-f envelopes shown in Fig. 7. (From reference no. 32.)

dependent of the effectiveness of the phase-focusing. For still higher levels of output power, the curves take on the form $V_{rf} = V_{max}(1 - e^{-\alpha t})$, indicating that bunching is practically complete.

While the above explanation appears reasonable, it is also evident that there is a great deal of room for further investigation on this subject.

3. "Strength" of Modes

In previous chapters, reference has been made to competition between modes, especially during build-up, and to the ability of a mode of oscillation to persist of high current and high power output. The strength of a mode may be considered to be a measure of its ability to persist either against possible competition from other modes, or against the destructive effect of excess anode voltage, which would tend to accelerate electrons to a velocity greater than that of the r-f wave. Evidently the principal factor determining the strength of a mode is simply the effectiveness of feedback, as might be expected for any feed-back oscillator. (No attempt will be made in this discussion to take into account the fact that the relative strengths of two modes might not necessarily be the same in the transient case as in steady-state conditions.)

The effectiveness of feed-back may be expressed in terms of the loop transmission in a feed-back amplifier. In such an amplifier, gain $= \frac{\mu}{1 - \mu\beta}$, where μ = gain without feed-back, and β represents the proportion of output signal which is fed back to the input. In an oscillator, for steady-state operation, $\mu\beta = 1$. When $\mu\beta$ is greater than unity, oscillations are building up. In usual feed-back oscillators, β depends upon the passive circuit and is constant. This may be taken to be the case for magnetrons. Then μ is dependent on amplitude of oscillation, and also depends on the loading of the resonant circuit. The feed-back ratio, β , depends upon anode geometry, and upon the geometry of the space between cathode and anode. It is quite important that bunching be effective near the cathode, but it is in this region that the r-f electric field is weakest, since the magnitude of the r-f radial electric field is approximately proportional to r^p, for the p^{th} order Hartree harmonic (cf. Chapter II). Hence, β is quite dependent on r_c/r_a. The dependence of stability on this ratio was pointed out by Slater in 1941,[(1)] and he estimated values of r_c/r_a as a function of N for π-mode operation, seeking a compromise between efficiency and stability.

For a high degree of stability, it is desirable that the feed-back signal be large, in order to be as insensi-

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(1) Reference No.4.

tive as possible to disturbances. This leads to a high value of $\mu\beta$ for small signals, and a relatively fast build-up should result from this. The effect of such a condition in terms of mode interactions will be discussed in more detail in subsequent chapters.

If only one Hartree component is considered in analyzing the magnetron as a feed-back oscillator, the analogy between the magnetron and a conventional feed-back oscillator is quite straightforward. The feed-back mechanisms in a magnetron oscillator with one r-f rotating wave have already been discussed. The assumption that the travelling waves corresponding to other Hartree harmonics pass by the electrons so quickly as to have no net effect has been made tacitly. The validity of this assumption is supported by the agreement between calculated and measured threshold voltages, since threshold voltages are calculated using such an assumption.

Under some circumstances, however, other Hartree harmonics cannot be safely ignored in considering the feed-back process. This is particularly true when the component to which the electron stream is coupled is of higher order than the lowest order component present. It has been pointed out before that the intensity of the radial r-f field component of a $p^{\underline{th}}$ order Hartree harmonic is approximately proportional to r^p. Thus, r-f field components which are of comparable magnitude near the anode may differ greatly near the cathode, with the advantage to the lowest-order component. An extreme example is illustrated in Fig. 9 by the detected r-f radial field pattern picked up by a rotating probe, displayed on a cathode-ray screen, as a function of angular position of the probe. The mode of

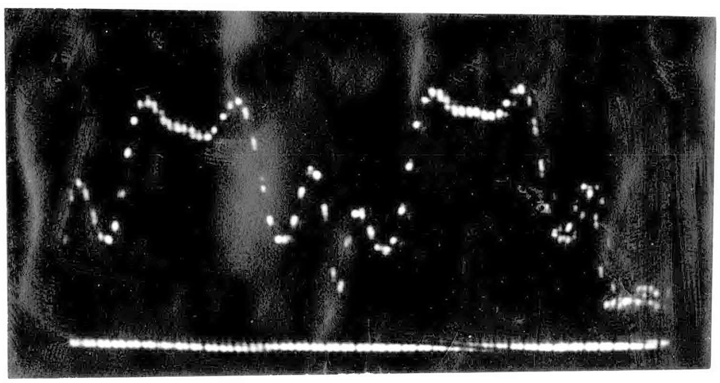

Figure 9. Radial r-f field pattern in model of AX-9 anode, as a function of angular position of rotating probe. (Reference No.12.)

resonance corresponds to n = 8 in an 18-segment rising-sun anode, but near the cathode the p = 1 component predominates in spite of the fact that at the anode the magnitude of the p = 8 component is larger. (It was mentioned in a previous chapter that possible orders of Hartree harmonics of the n^{th} mode are given by $p = n + \nu \frac{N}{2}$ in rising-sun anodes, where ν is a positive or negative integer or zero.) The result is that the rejection of unfavorable electrons by the desired component is seriously disrupted by the presence of a much larger component of the same mode. The 8/8/18 mode has never been observed in magnetrons having anode structure similar to the one which produced the field pattern of Fig. 9,(1) nor has the lower-voltage 10/8/18 mode, corresponding to a higher-order harmonic of the same resonance, been observed.

A factor which must not be neglected in considering the strength of a mode of oscillation is the loading of the system, including both power to the load and circuit losses. In general, lighter loading leads to greater r-f amplitudes, and greater r-f amplitude leads to a system less easily disturbed by anything outside of that mode of oscillation.

4. Mode Failure in Absence of Other Modes

Failure of oscillation in magnetrons, as anode voltage and current are raised, may, in general, be considered to be the result of failure of the feed-back mechanisms. The

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(1) The magnetrons referred to here are the MF-series, which have been wavelength scaled to 10.7 cm from the 3.16 cm AX-9 (Columbia Radiation Laboratory). The MF-series are under development at the Research Laboratory of Electronics, M.I.T.

fundamental causes of such failure appear to be first, the inability of the r-f field to maintain synchronism between itself and the electrons; second, competition from other modes of oscillation; and third, the flow of d-c current to the anode when d-c cutoff is reached, where current can flow to the anode in absence of r-f field. There are secondary factors which contribute to the first two primary causes of failure, but only the three named here seem fundamental.

In this section only the first of the above causes will be considered. The second will be discussed in detail in subsequent chapters. The third cause for failure of oscillation is rarely met with, and it is evident that nothing can be done about it, except to increase magnetic field, and thus increase the value of d-c cutoff voltage.

Stated briefly, what must happen in the first case is that the r-f radial field is no longer strong enough to keep electrons in synchronism with itself, and the feedback mechanism becomes seriously impaired. As the anode voltage is increased in an oscillating magnetron, the tendency to pull electrons ahead of synchronous velocity also increases. The electrons now tend to lead the point in the rotating wave at which maximum energy would be given up, and their effectiveness becomes diminished. Eventually the combination of excessive acceleration of the d-c field, combined with the decreased effectiveness of the electrons in building up the r-f field, leads to failure of oscillations.

However, the increase in anode voltage also leads to increased amplitude of oscillation and, therefore, the phase-focusing action may increase at a rate comparable with or greater than the increasing tendency of the d-c field to make electrons exceed synchronous velocity. Another effect of the increased r-f amplitude is that the transit time of an electron between cathode and anode is shortened. This has the effect of decreasing the electronic efficiency (detrimental to stability), but it also decreases the time during which a favorable electron can get out of phase.

Another requirement placed upon the phase-focusing mechanism is that it prevent dispersion of electron "spokes," which would otherwise take place as a result of the mutual repulsion of electrons. Little information is available as to how charge density within the spokes varies with changes in operating conditions. If, at higher power levels, the density is higher, the de-bunching effect due to mutual repulsion is stronger. This effect will increase the tendency toward instability. However, at higher power levels, greater de-bunching should be opposed by greater r-f field intensity.

A secondary cause of failure is inadequate emission. In order to get an increase of r-f amplitude necessary to maintain stability in the fact of increased d-c voltage, an increase in current is necessary. If the cathode cannot supply the necessary current, the collapse of oscillation then results from deficient phase focusing. The cathode also produces more subtle effects, since the distribution of electric fields may depend on whether emission is space-charge limited, and since the emission velocities may also affect the distribution of electric fields. The relative importance of the various effects produced by the cathode is not clear. The most obvious of the cathode effects, that is, the presence of sufficient emission, is evidently inadequate to explain all observed phenomena, as has been pointed out by Dench, of Raytheon, and reported by Welch of the University of Michigan.[1] For low values of temperature-limited emission (from an oxide cathode), the maximum anode current for stable π-mode oscillation increased with increasing temperature; for higher values of emission, a maximum was reached, after which there was a slight decrease in maximum current with increasing temperature.

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(1) Reference No.35, pp.37-44.

The fundamental reasons for instability, described above, remain the same in principle when there are significant non-uniformities in the magnetron in an axial direction. It is pointed out by Feldmeier[(1)] that uniformity of magnetic field is necessary because of the large variation in current which can result from small variations in magnetic field. Rieke and Fletcher[(2)] suggested a change in the pole piece design of the 2J39 magnetron which led to a more uniform magnetic field, and a great increase in stability was achieved. (Experimental work on the 2J39 will be described in a later chapter.) It seems reasonable to attribute the poor stability of magnetrons with non-uniform magnetic fields to the fact that when one portion of the anode is drawing a large current, another portion (where the magnetic field is stronger) may be drawing little or none; and when the anode voltage is raised to give the latter portion of the anode a chance to draw a reasonable amount of current, the electrons in the portion of the interaction space where magnetic field is weak may well find themselves out of synchronism, and therefore not contributing their share of energy to the r-f wave. If the velocity of the electrons in the weak magnetic field is near that corresponding to another mode, it is not inconceivable that com-

(1) Reference No.1, Chapter 13 (by J.R.Feldmeier), p.552.
(2) Reference No.17; also Reference No.1, p.380.

petition will take place, with the second attempting to build up and suppress the first mode.

Axial non-uniformities in the r-f field are also possible. These are most prevalent in strapped anodes(1) and in closed-end rising-sun anodes.(2) The line of reasoning in the preceding paragraph suggests that these non-uniformities affect both the stability and efficiency adversely. For example, it is to be expected that synchronism between r-f field and electrons will be lost in a region of weak r-f field much more readily than where the field is strong. However, the extent of these effects has apparently not been studied, and there seems to be no evidence to indicate that it is very serious, especially in strapped magnetrons.

There has been one kind of axial non-uniformity which has been used to improve operation. It has been found that a slight enlargement of the cathode for a small distance at each end helped to prevent misfiring without adversely affecting efficiency or stability to any great extent.(3)

The preceding discussion has indicated that the two principal factors which are most important to stable operation are axial uniformity (especially magnetic) and the ef-

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(1) Reference No.18, p.17.
(2) Reference No.1, Chapter 3 (by N.Kroll), p.110.
(3) Reference No.1, Chapter 8 (by F.F.Rieke), p.379.

fectiveness of the r-f feed-back. The latter depends principally on the intensity of the r-f electric field and on its freedom from disturbances by other modes and by components of the desired mode other than the one to which the electrons are coupled. Disturbance by another mode may take place when both the wanted and unwanted modes are near each other in frequency, and this may be prevented by adequate mode separation, as described in Chapter II. Effective bunching of electrons by an r-f electric field tends to suppress any tendency of the electron stream to supply energy to other modes, and this will be discussed in detail in later chapters. Unwanted Hartree harmonics of the desired mode ordinarily do not cause trouble in π-mode oscillation, with the exception of zero-order component interference in rising-sun magnetrons.(1) The latter type of interference, it has been found, can be effectively eliminated by proper choice of magnetic field.

There are two principal means by which the r-f feed-back can be made more intense. The first is to lighten loading. Thus, a given amount of r-f power leads to larger r-f electric fields, but such a change also leads to lower efficiency, both electronic and in the r-f circuit. The second means for increasing the r-f feed-back is to alter the

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(1) Reference no. 1, Chapter III (by N. Kroll), pp. 98-100.

geometry of the system, especially by increasing r_c/r_a in order to increase the r-f intensity near the cathode. (The latter change was also discussed in Chapter II.) This change also leads to lower electronic efficiency. Thus, in these two instances, it is necessary to sacrifice efficiency in order to gain stability, and magnetron design becomes a compromise.

CHAPTER IV

MODE INTERACTIONS: NON-LINEAR CIRCUIT THEORY

1. Non-Linear Triode Oscillator

The fundamental theory on which this chapter is based was first studied by van der Pol about thirty years ago.(1) The earliest theory was for a simple non-linear triode oscillator, and later the theory was extended to cover non-linear triode oscillators with "two degrees of freedom," that is, two modes in the resonant circuit.

For the triode oscillator operating with a simple resonant circuit, as in Fig. 10, it was assumed that the relationship between instantaneous voltage, v, across the resonant circuit, and the instantaneous current, i, through the triode, may be represented by $i = \psi(v)$. The differential equation expressing the performance of a circuit with such a triode operating with an RLC parallel resonant circuit (cf. Fig. 10) is:

$$C \frac{d^2 v}{dt^2} + \frac{d}{dt}\left(\frac{v}{R} + \psi(v)\right) + \frac{v}{L} = 0 \tag{1}$$

This is one form of what has become known as van der Pol's equation.

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(1) References No.23 and 24.

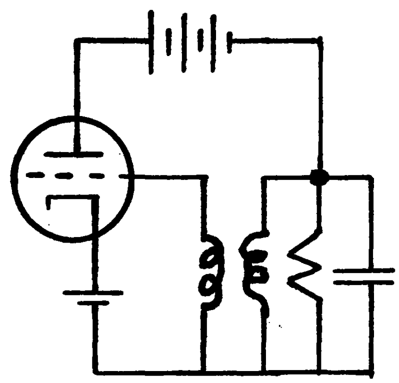

Figure 10. Elementary feed-back oscillator.

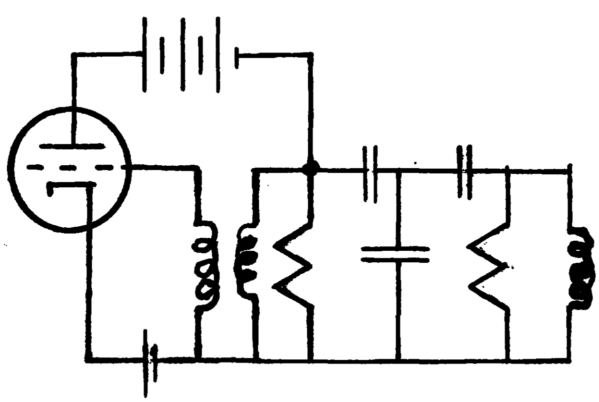

Figure 11. Triode oscillator with two degrees of freedom. (From reference no. 23.)

The analysis can be advanced further by representing $\psi(v)$ as a polynomial containing first and third powers of v. It is shown by van der Pol that even powers in the expression for $\psi(v)$ have a negligible effect if the resonant system is high-Q. (This will be brought out in detail in subsequent discussion in this chapter.) If $\psi(v)$ is of the form:

$$\psi(v) = av - bv^3 \tag{2}$$

then equation (1) can be reduced to the following form, normalized in terms of final magnitude of the oscillations:[1]

$$\ddot{v} - \alpha(1 - v^2)\dot{v} + \omega^2 v = 0 \qquad \varepsilon = \frac{\alpha}{\omega} \ll 1 \tag{3}$$

When there is a resonant circuit with two possible modes of resonance, as in Fig. 11, there are two simultaneous equations[2] derived in the same manner as equation (3):

$$\ddot{v}_1 - \alpha_1(1 - v_1^2)\dot{v} + \omega_1^2 v_1 + k_1\omega_1^2 v_2 = 0 \qquad \varepsilon_1 = \frac{\alpha_1}{\omega_1} \ll 1 \tag{4a}$$

$$\ddot{v}_2 + \alpha_2 \dot{v}_2 + \omega_2^2 v_2 + k_2 \omega_2^2 v_1 = 0 \qquad \varepsilon_2 = \frac{\alpha_2}{\omega_2} \ll 1 \tag{4b}$$

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(1) Reference No.24, p.1052.
(2) Reference No.24, p.1053.

In (4a) and (4b), the coupling coefficient between the two resonant branches of the circuit is **k**, where $k^2 = k_1 k_2$. In finding an approximate solution to these simultaneous equations, two possible frequencies of oscillation were found, and designated ω_I and ω_{II}. Therefore, the system may be described as having two possible modes of oscillation.

It will be shown later in this chapter how this kind of analysis can be applied to a magnetron with two possible modes of oscillation.

2. Magnetron Oscillator: One Mode

When conditions in a magnetron are such that oscillation in only one mode is possible, equation (1) may be used to represent the build-up of oscillations. The means of feed-back discussed in Chapter III now replace the grid circuit of Fig. 10. If it is assumed as a rough approximation that equation (2) can be applied to the non-linear characteristics of a magnetron, then equation (3) replaces (1), and an approximate solution can be obtained. Assume the following solution of (3) for **v**:

$$v = A \cos (\omega t - \phi) = A \cos u \qquad (5)$$

In (5), A and ϕ may be functions of time. Differentiating:

$$\dot{v} = \dot{A} \cos u - A\dot{u} \sin u \qquad (6)$$

and:

$$\ddot{v} = \ddot{A} \cos u - 2 \dot{A} \dot{u} \sin u - A \dot{u}^2 \cos u + A \ddot{u} \sin u \qquad (7)$$

Since $u = \omega t - \phi$, then $\dot{u} = \omega - \frac{d\phi}{dt}$, and $u = -\frac{d^2\phi}{dt^2}$.

In order to find $\dot{v}v^2$, which appears in (3), the quantity v^3 may be found and then differentiated, since $\frac{1}{3}\frac{d}{dt}\, v^3 = \dot{v}v^2$:

$$v^3 = A^3\left(\tfrac{3}{4}\cos u + \tfrac{1}{4}\cos 3u\right) \qquad (8)$$

$$\dot{v}v^2 = \frac{1}{3}\frac{d}{dt}v^3 = A^3\dot{u}\left(-\tfrac{1}{4}\sin u - \tfrac{1}{4}\sin 3u\right) + A^2\dot{A}\left(\tfrac{3}{4}\cos u + \tfrac{1}{4}\cos 3u\right) \qquad (9)$$

If the resonant circuit is high-Q, any component of current at frequency $3u$ (which is approximately $3\omega t$) will give rise to a very small voltage, and will therefore be neglected. At this point in the discussion, it seems appropriate to point out that any v^2 terms, which might appear in $\psi(v)=i$, lead only to a component of current at frequency $2u$ and a d-c component, neither of which should generate any appreciable voltage across the resonant circuit.

Now, substituting (5), (6), (7), and (9) into (3), an equation is obtained containing terms in $\sin u$ and in $\cos u$. If the terms containing $\sin u$ are equated, and the result divided by $\sin u$:

$$-2\dot{A}\dot{u} + A\ddot{u} + \alpha A\dot{u} - \tfrac{1}{4}\alpha A^3\dot{u} = 0 \qquad (10)$$

If terms containing cos u are equated, and divided by cos u:

$$\ddot{A} - A\dot{u}^2 - \alpha\dot{A} + \tfrac{3}{4}\alpha A^2\dot{A} + \omega^2 A = 0 \tag{11}$$

In a high-Q system, such as a magnetron, the first and third terms of equation (3) are very large as compared with the second. Therefore, the quantity u is very nearly equal to ω, and $\frac{d\phi}{dt}$ is much less than ω. In order to simplify the solution of (10) for $\dot{A}$, $\ddot{u}$ $(= \frac{d^2\phi}{dt^2})$ will be neglected, and then (10) can be divided by $\dot{u}$:

$$\dot{A} = \tfrac{1}{2}\alpha A(1 - \tfrac{1}{4}A^2) \tag{12}$$

It is possible to integrate (12) and find an explicit solution for αt in terms of A:

$$\alpha\, dt = \frac{8\,dA}{A(4 - A^2)} \tag{13}$$

$$\alpha t = \ln \frac{A^2}{4 - A^2} + C \tag{14}$$

Building-up of oscillations must proceed from some finite magnitude of voltage. Let $A = A_0$ when $t = 0$. Then equation (14) becomes:

$$\alpha t = \ln \frac{A^2(4 - A_0^2)}{A_0^2(4 - A^2)} \tag{15}$$

The solution to the build-up equation given by (15) is shown in Fig. 12, in which A is plotted as a function of αt.

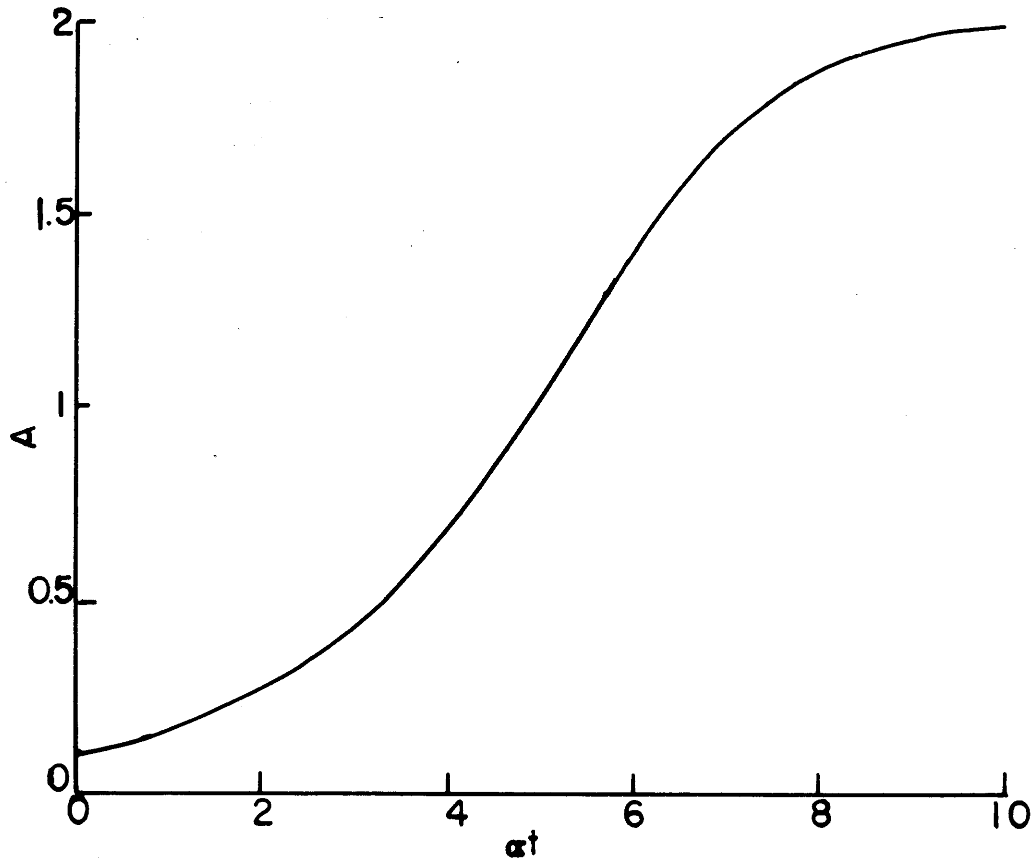

Fig.12. Approximate solution of van der Pol's equation, where the solution is assumed to be of the form, A cos $(\omega t - \phi)$, and ω is much greater than dA/dt. The actual form of van der Pol's equation solved here is expressed by equation (3).

On the basis of the results already obtained, it is possible to estimate the effect of neglecting $A\ddot{u}$ in equation (10) by solving (11) for $\frac{d\phi}{dt}$. First, in order to find $\ddot{A}$, it is necessary to differentiate (12):

$$\ddot{A} = \frac{1}{2}\alpha \dot{A}\left(1 - \frac{3}{4}A^2\right)$$

$$= \frac{1}{4}\alpha^2 A\left(1 - \frac{1}{4}A^2\right)\left(1 - \frac{3}{4}A^2\right) \tag{16}$$

Substituting into (11):

$$\frac{1}{4}\alpha^2 A\left(1 - \frac{1}{4}A^2\right)\left(1 - \frac{3}{4}A^2\right) - \alpha\dot{A}\left(1 - \frac{3}{4}A^2\right) + \left(\omega^2 - \dot{u}^2\right)A = 0 \tag{17}$$

$$\dot{u}^2 = \omega^2 - 2\omega\frac{d\phi}{dt} + \left(\frac{d\phi}{dt}\right)^2 \tag{18}$$

The term $\left(\frac{d\phi}{dt}\right)^2$ will be neglected, since $\frac{d\phi}{dt}$ is much smaller than ω. Therefore, substituting the results of (12) and of (18) into equation (17), we find:

$$2\omega A\frac{d\phi}{dt} = \frac{1}{4}\alpha^2 A\left(1 - \frac{1}{4}A^2\right)\left(1 - \frac{3}{4}A^2\right)$$

or,
$$\frac{d\phi}{dt} = \frac{1}{8}\frac{\alpha^2}{\omega}A\left(1 - A^2 + \frac{3}{16}A^4\right) \tag{19}$$

This expression leads to the deviation of frequency during build-up from the normal value ω. It is not very reliable as applied to a magnetron because of the reactance characteristics of the electron stream. Furthermore, it is quite small, because α is much smaller than ω in a high-Q system.

A value of $\ddot{u}$ to use in (10) may be found by differentiating (19):

$$\frac{d^2\phi}{dt^2} = \ddot{u} = \frac{1}{4}\frac{\alpha^2}{\omega}A\left(1-\frac{1}{4}A^2\right)\left(-2A+\frac{3}{4}A^3\right) \quad (20)$$

Since A has an order of magnitude not greater than that of unity, the order of magnitude of $\frac{d^2\phi}{dt^2}$ is $\frac{\alpha^3}{\omega}$, and is therefore small as compared with the other terms of (10) which have the order of magnitude of $\underline{\alpha\dot{u}}$ ($\approx \alpha\omega$).

In order for oscillation to build up and become stable, it is necessary that the signs in equation (2) be as they are given there, or in other words, that _a_ and _b_ are positive. If _a_ is not positive, the solution to (3) does not correspond to the possible build-up of small-amplitude oscillations. If _b_ is not positive, there will be no limit to the magnitude of oscillations.

3. Magnetron with Two Modes of Operation

The equivalent circuit for a resonant cavity with more than one mode of resonance, and with each resonant mode coupled, to a greater or less degree, to a single load, is shown in Fig. 13a. The circuit which will be considered in investigating the interaction of two modes in a magnetron is shown in Fig. 13b. It is assumed that the magnetron is a high-Q system, and that resonances are well separated in frequency, so that the impedance of one of the parallel-resonant circuits is negligible at the resonant frequency of the other. Therefore, any interaction in the passive circuit may be neglected, and the simpler circuit of Fig.13b will be studied.

First, we shall assume that the voltage developed across the circuit in Fig.13b may be represented by:

$$v = V_1 e^{\sigma_1 t} \cos \omega_1 t + V_2 e^{\sigma_2 t} \cos \omega_2 t \tag{21}$$

$$v = \mathcal{R}\left(V_1 e^{\lambda_1 t} + V_2 e^{\lambda_2 t}\right) \qquad \begin{aligned} \lambda_1 &= \sigma_1 + j\omega_1 \\ \lambda_2 &= \sigma_2 + j\omega_2 \end{aligned} \tag{22}$$

The phase of each frequency component has not been generalized (i.e., the voltage has not been expressed in the form $V_1 \cos(\omega_1 t + \phi)$), because the solution which will be found here will not depend on the phase relationship, pro-

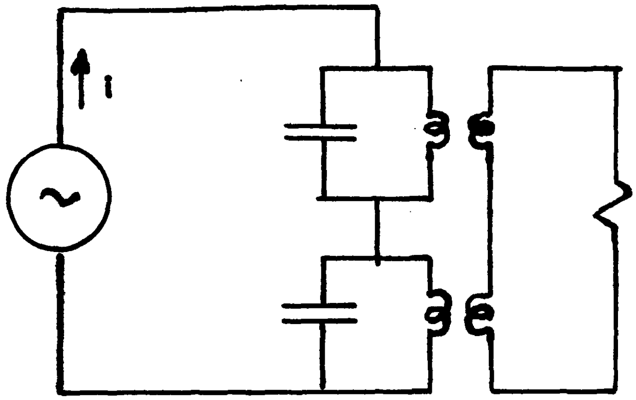

Figure 13a. Equivalent circuit for magnetron oscillator with two modes in resonant circuit.

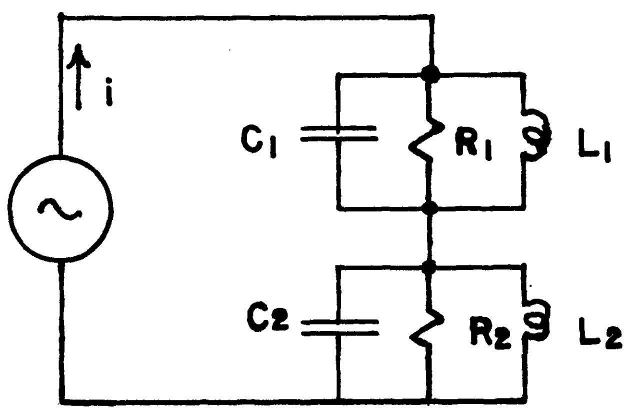

Figure 13b. Simplified equivalent cricuit for magnetron oscillator with two modes in resonant circuit.

vided ω_1/ω_2 can not be represented by a rational fraction.

Unless indicated otherwise, in all of the equations which follow involving complex quantities, it will be implied that it is the real part which is significant. Therefore (22) may be rewritten simply:

$$v = V_1 e^{\lambda_1 t} + V_2 e^{\lambda_2 t} \tag{23}$$

Likewise:

$$i = I_1 e^{\lambda_1 t} + I_2 e^{\lambda_2 t} \tag{24}$$

It is assumed further that λ_1 is approximately equal to $j\frac{1}{\sqrt{C_1 L_1}}$, and that λ_2 is approximately equal to $j\frac{1}{\sqrt{C_2 L_2}}$.

Now let Z_1 represent the impedance across the L_1-C_1-R_1 part of the circuit, and Z_2 the impedance across the L_2-C_2-R_2 part. Then $V_1 \approx I_1 Z_1$ and $V_2 \approx I_2 Z_2$. These expressions result from the fact that $Z_1(\lambda_1)$ is large but $Z_1(\lambda_2)$ is small, and $Z_2(\lambda_2)$ is large while $Z_2(\lambda_1)$ is small. Then:

$$I_1 = V_1 \left(\lambda_1 C_1 + \frac{1}{R_1} + \frac{1}{\lambda_1 L_1} \right) \tag{25}$$

$$I_2 = V_2 \left(\lambda_2 C_2 + \frac{1}{R_2} + \frac{1}{\lambda_2 L_2} \right) \tag{26}$$

In the equivalent circuit, the electron stream is represented by a single current source, and the voltage fed back is proportional to $(V_1 + V_2)$. The current is therefore a non-linear function of the sum of the two voltages.

Now, as in the preceding section, let $i = av - bv^3$. In analyzing the effect of the cubic term, the exponential representation is no longer valid. Therefore, from equation (21):

$$\begin{aligned} v^3 = V_1^{\,3} e^{3\sigma_1 t} \cos^3 \omega_1 t &+ 3 V_1^{\,2} V_2 e^{2\sigma_1 t} e^{\sigma_2 t} \cos^2 \omega_1 t \cos \omega_2 t \\ &+ 3 V_1 V_2^{\,2} e^{\sigma_1 t} e^{2\sigma_2 t} \cos \omega_1 t \cos^2 \omega_2 t \\ &+ V_2^{\,3} e^{3\sigma_2 t} \cos^3 \omega_2 t \end{aligned} \tag{27}$$

$$\cos^3 \omega_1 t = \tfrac{3}{4} \cos \omega_1 t + \tfrac{1}{4} \cos 3\omega_1 t \tag{28a}$$

$$\cos^3 \omega_2 t = \tfrac{3}{4} \cos \omega_2 t + \tfrac{1}{4} \cos 3\omega_2 t \tag{28b}$$

(cf. eq.8)

$$\cos^2 \omega_1 t \cos \omega_2 t = \tfrac{1}{2} \cos \omega_2 t + \tfrac{1}{2} \cos \omega_2 t \cos 2\omega_1 t \tag{29a}$$

$$\cos \omega_1 t \cos^2 \omega_2 t = \tfrac{1}{2} \cos \omega_1 t + \tfrac{1}{2} \cos \omega_1 t \cos 2\omega_2 t \tag{29b}$$

It will be assumed that all current components other than those at frequencies ω_1 and ω_2 will lead to negligible voltage components across the resonant circuits. The products, $\cos \omega_1 t \cos 2\omega_2 t$ and $\cos 2\omega_1 t \cos \omega_2 t$, lead to frequency components of $(2\omega_2 \pm \omega_1)$ and $(2\omega_1 \pm \omega_2)$, respectively. Therefore, v^3 will be approximated by:

$$v^3 = \tfrac{3}{4} V_1^3 e^{3\sigma_1 t} \cos \omega_1 t + \tfrac{3}{2} V_1^2 V_2 e^{2\sigma_1 t} e^{\sigma_2 t} \cos \omega_2 t$$
$$+ \tfrac{3}{2} V_1 V_2^2 e^{\sigma_1 t} e^{2\sigma_2 t} \cos \omega_1 t$$
$$+ \tfrac{3}{4} V_2^3 e^{3\sigma_2 t} \cos \omega_2 t \qquad (30)$$

Now if $i = I_1 e^{\lambda_1 t} + I_2 e^{\lambda_2 t}$, then:

$$I_1 = \left\{ a V_1 - \tfrac{3}{4} b V_1 \left(V_1^2 e^{2\sigma_1 t} + 2 V_2^2 e^{2\sigma_2 t} \right) \right\} \qquad (31)$$

$$I_2 = a V_2 - \tfrac{3}{4} b V_2 \left(V_2^2 e^{2\sigma_2 t} + 2 V_1^2 e^{2\sigma_1 t} \right) \qquad (32)$$

Now, current in the active part of the circuit must equal current in the passive circuit. Therefore (31) must be equated to (25), and (32) to (26). But first, (18) can be simplified in the following manner:

$$\lambda_1 C_1 + \frac{1}{\lambda_1 L_1} = \frac{C_1}{\lambda_1} \left(\lambda_1^2 + \frac{1}{L_1 C_1} \right) \qquad (33)$$

If $\omega_1 = \dfrac{1}{\sqrt{L_1 C_1}}$, then:

$$\lambda_1 C_1 + \frac{1}{\lambda_1 L_1} = \frac{C_1}{\lambda_1} \left(\lambda_1^{\,2} + \omega_1^{\,2} \right) \tag{34}$$

$$= \frac{C_1}{\sigma_1 + j\omega_1} \left(\sigma_1^{\,2} + 2j\omega_1\sigma_1 - \omega_1^{\,2} + \omega_1^{\,2} \right)$$

Since ω_1 is much greater than σ_1, then:

$$\lambda_1 C_1 + \frac{1}{\lambda_1 L_1} \approx 2C_1\sigma_1 \tag{35}$$

It is the real part which is of greatest interest here. The small imaginary part of (34), which has been neglected, would lead to only a very small change in frequency of the kind expressed in the previous section by $\frac{d\phi}{dt}$ (cf. equation 19).

Therefore, equating (31) to (25), and dividing by V_1:

$$2C_1\sigma_1 + \frac{1}{R_1} = a - \frac{3}{4} b \left(V_1^{\,2} e^{2\sigma_1 t} + 2V_2^{\,2} e^{2\sigma_2 t} \right) \tag{36}$$

Now let $\alpha_1 = a - \frac{1}{R_1}$, $\mathcal{V}_1 = V_1 e^{\sigma_1 t}$, and $\mathcal{V}_2 = V_2 e^{\sigma_2 t}$.

In other words, at any time t, $\mathcal{V}_1$ and $\mathcal{V}_2$ represent the respective magnitudes of voltage developed at frequencies ω_1 and ω_2. Then:

$$\sigma_1 = \frac{\alpha_1}{2C_1} - \frac{3b}{8C_1}\left(V_1^2 + 2V_2^2\right)$$
$$= \frac{3b}{8C_1}\left(\frac{4}{3}\frac{\alpha_1}{b} - V_1^2 - 2V_2^2\right) \tag{37}$$

A new parameter, $V_I^2 = \frac{4a_1}{3b}$ can be introduced. V_I is the magnitude that V_1 will approach in steady state ($\frac{dV_1}{dt} = \frac{dV_2}{dt} = 0$) if $V_2 = 0$. Therefore:

$$\sigma_1 = \frac{3b}{8C_1}\left(V_I^2 - V_1^2 - 2V_2^2\right) \tag{38}$$

By symmetry:

$$\sigma_2 = \frac{3b}{8C_2}\left(V_{II}^2 - V_2^2 - 2V_1^2\right) \tag{39}$$

where $V_{II}^2 = \frac{4a_2}{3b}$.

Thus, it is seen from equations (38) and (39) that the magnitude of oscillations at each frequency affects the rate of build-up at the other frequency, or in other words, oscillation in one mode reduces the rate of build-up in other modes. Furthermore, it is very significant that, according to this analysis, <u>the rate of build-up in one mode is affected less by the magnitude of its own oscillation than by the magnitude of oscillation in the other mode</u>. This condition is one which Rieke assumed,(1) at the same time stating that it was open to question.

It is also very significant to point out that, although van der Pol did not use quite the same method of solution as given above, his approximate solution for the circuit of Fig. 11 led to the same results as in equations (38) and (39). Furthermore, van der Pol did not assume either that coupling between the two resonant sections was small, nor that their resonant frequencies were separated by any large amount.

The results in (38) and (39) may be plotted, as in Fig. 14. Here, the coordinate axes correspond to $\mathcal{V}_1^2$ and $\mathcal{V}_2^2$, respectively. Lines of constant σ_1 and σ_2 are drawn, and these are shown in order to separate the plane into regions according to which modes are building up or decaying. Region I, which includes portions of both axes, corresponds to $\sigma_1 > 0$ and $\sigma_2 > 0$. Therefore, if the state of oscillation is such that the magnitudes in each mode, when plotted in the plane, line in region I, then both modes are building up. In region II, $\mathcal{V}_1^2$ is increasing, $\mathcal{V}_2^2$ decreasing; in region III, $\mathcal{V}_1^2$ is decreasing, $\mathcal{V}_2^2$ increasing; and in region IV, both are decreasing.

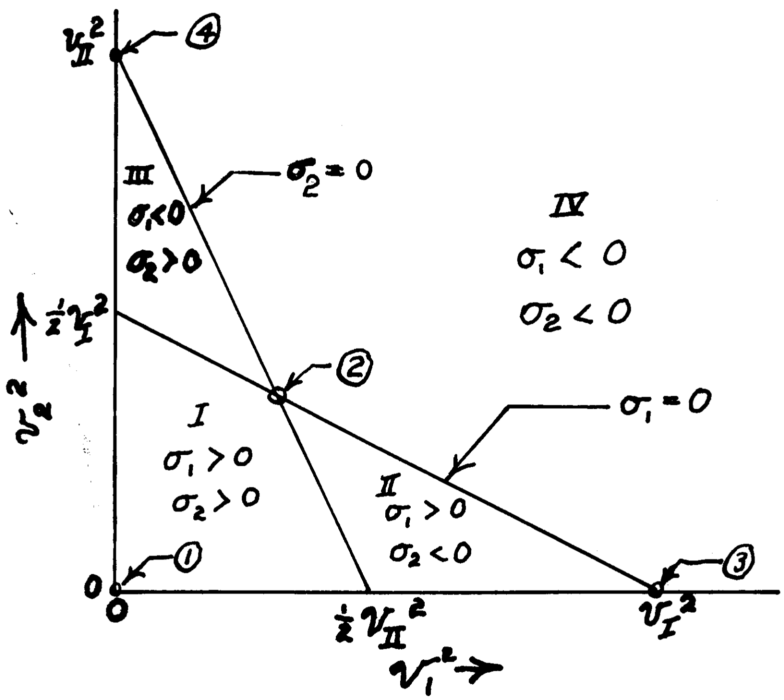

Figure 14a. Plot of equations (38) and (39)--see text--to show areas of build-up and decay of each mode as functions of $\mathcal{V}_1^2$ and $\mathcal{V}_2^2$. Either mode may be stable here.

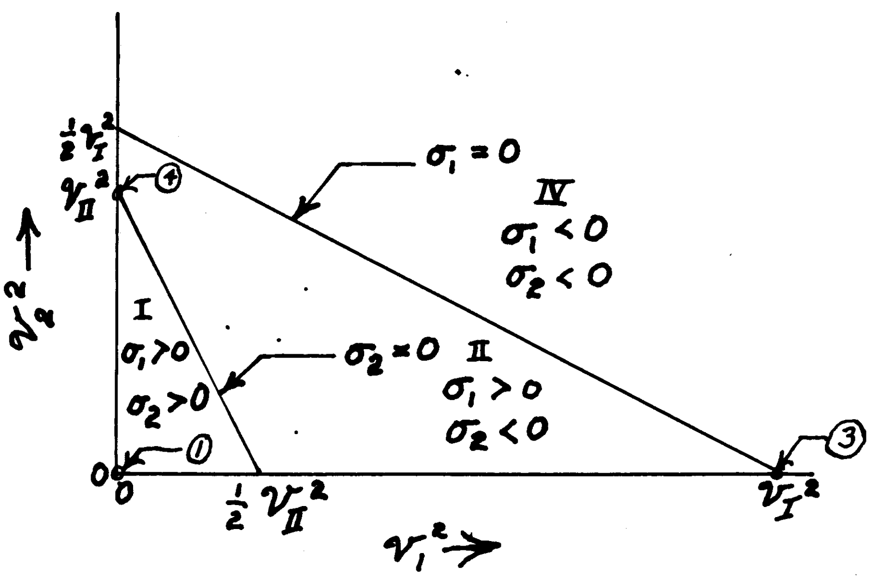

Figure 14b. Same as Fig. 14a, except that here only one mode ($\mathcal{V}_1^2 = \mathcal{V}_I^2$) can be stable.

In the references mentioned,[1,2] van der Pol has examined the results for "possible" solutions and stable solutions among the "possible" solutions. Possible solutions were defined as those for which $\frac{d\mathcal{V}_1}{dt} = \frac{d\mathcal{V}_2}{dt} = 0$. There are four, as shown by the encircled Arabic numerals in Fig.14(a). Of these, he finds that only two are stable. Solution (1), where $\mathcal{V}_1 = \mathcal{V}_2 = 0$, is unstable because both σ_1 and σ_2 are greater than zero, and any small disturbance, such as shot noise, will start the building-up of both modes. Solution (2) represents the condition where $\sigma_1 = \sigma_2 = 0$, and where $\mathcal{V}_1^2 = \frac{1}{3}(2\mathcal{V}_{II}^2 - \mathcal{V}_I^2)$, and $\mathcal{V}_2^2 = \frac{1}{3}(2\mathcal{V}_I^2 - \mathcal{V}_{II}^2)$. The instability here is more subtle than in solution (1). However, it may be shown that if the respective values of $\mathcal{V}_1^2$ and $\mathcal{V}_2^2$ are altered slightly so as to get into region II, the build up of $\mathcal{V}_1^2$ will continue, together with decay of $\mathcal{V}_2^2$, and the operating point will move away from point (2). Any disturbance into region III will also lead to the moving away from point (2) by the point representing actual operating conditions. Furthermore, any excursion of the operating point into region I or region IV will in general be followed by movement of the point into regions II or III, rather than back to point (2), and conditions will proceed away from that point.

Points (3) and (4) are similar to each other in character. For example, point (3) represents a condition where $\mathcal{V}_1^{\,2} = \mathcal{V}_I^{\,2}$, $\sigma_1 = 0$, $\mathcal{V}_2^{\,2} = 0$, and $\sigma_2 < 0$. Therefore $\mathcal{V}_1$ is both stationary in magnitude (because $\sigma_1 = 0$) and stable (because any disturbance of $\mathcal{V}_1^{\,2}$ from this position will change conditions so as to cause it to return). At point (3), $\mathcal{V}_2$ is also both stationary and stable, because $\frac{d\mathcal{V}_2}{dt} = 0$, and $\sigma_2 < 0$. Thus any oscillation which might appear corresponding to frequency ω_2 would be quickly damped out.

For similar reasons, at point (4), $\mathcal{V}_1^{\,2}$ and $\mathcal{V}_2^{\,2}$ are both stationary and stable, with $\mathcal{V}_1^{\,2} = 0$ and $\sigma_1 < 0$ here.

Another possible set of conditions may give rise to a set of build-up characteristics,which can be represented by Fig. 14(b). Here, solution (2) and region III of Fig. 14(a) are absent. But what is much more important is that at point (4), σ_1 is still positive. Therefore, even if this point could be reached, with $\mathcal{V}_2^{\,2}$ at a stationary value, the value of $\mathcal{V}_1^{\,2} = 0$ is unstable, and any disturbance would cause it to build up, and the operating point would proceed into a region in which $\mathcal{V}_2^{\,2}$ must decay. Therefore, only point (3) is both stationary and stable. This situation is characterized by the fact that $\frac{1}{2}\,\mathcal{V}_I^{\,2} > \mathcal{V}_{II}^{\,2}$. A comparable situation, where the only stable oscillation which can take place corresponds to $v_2^2 = v_{II}^2$, is given by $\frac{1}{2} v_{II}^2 > v_I^2$.

The expression, oscillation hysteresis, was used by van der Pol to express what happened as relative conditions for the two modes were changed continuously, during which time oscillation was maintained. Suppose oscillation were started under conditions shown by Fig. 14(b). Oscillation takes place with v_1 suppressing v_2. Then if the relative strength of the two modes is changed (see Chapter III) so that conditions of Fig. 14(a) prevail, the oscillating mode will still correspond to v_1. Only when $\frac{1}{2} v_{II}^2$ becomes greater than v_I^2 will v_2 build up and suppress v_1. Now beginning from the latter condition, with v_2 present and $v_1 = 0$, let the conditions be gradually changed in the opposite direction. This time the mode change point does not correspond to $\frac{1}{2} v_{II}^2 = v_I^2$, as before, but instead to $v_{II}^2 = \frac{1}{2} v_I^2$. It was this effect that was called oscillation hysteresis. It may be described, in other words, by saying that the existing mode tends to persist.

Now the applicability of such an analysis to the magnetron should be examined. The feed-back mechanisms of a magnetron were discussed in Chapter III. The effects are easily shown, both experimentally and theoretically, to be non-linear. Furthermore, it is certainly true that, although the r-f fields corresponding to various modes of oscillation may be superimposed in the passive circuit, nevertheless, they must act together on the same non-linear electron system, and thus interaction must be present. A difference between magnetrons and conventional triodes is that essentially, the triode performance is representable by a single lumped element, whereas feed-back in the magnetron is distributed throughout the electron interaction space, for which the transit time is of the order of cycles instead of a small fraction of a cycle; however, the summation of effects in the magnetron can lead to a system representable by some mathematical expression comparable to that for a triode. In a magnetron, and in a triode circuit as well, it is not necessarily true that the feed-back ratio is the same for both resonances (as it was taken to be here); if it is not the same, this can be remedied in the equivalent circuit by using a new equivalent circuit where, for example, $\sqrt{L_1/C_1}$ is changed, without changing the resonant frequency, which is determined by $\sqrt{L_1C_1}$. By this means, the original current output at the original frequency will lead to a different fed-back voltage. It is also not necessarily true in a magnetron that the non-linearities will be similar for the two modes, that is, that a and b (i.e., in the active part of the circuit, $i = av - bv^3$ was assumed) will bear the same ratio to each other; but this ratio was effectively destroyed in equation (30), where α_1 enters the picture instead of a, because in general, α_1 $(= a - \frac{1}{R_1})$ is not equal to α_2 $(= a - \frac{1}{R_2})$.

The parts of the above discussion which seem most open to question are the representation of the non-linear portion of the system, first by a non-linear circuit element in which the output current is always in phase with the control voltage, and second by the very elementary form of the non-linear expression. The author has not undertaken the analysis of a non-linear system in which the phase of the fed-back signal varies. The analysis of non-linear circuits with somewhat different characteristics than those discussed here will be taken up in the next section. It is also true that the non-linear characteristics of any electron device depend upon external conditions-applied voltages, magnetic field (for a magnetron), etc. But, in general, the external conditions may be affected by the state of oscillation. Therefore, the interaction between external effects and active circuit characteristics must be taken into account before a complete picture of the problem of non-linear systems can be obtained.

4. Non-Linear Oscillators with More General Non-Linear Characteristics: One Mode

One of the most significant features of the non-linear theory developed so far has been the changing of an instantaneous function of voltage into another function which expresses the equivalent voltage magnitudes in the sinusoidal case, assuming that only the fundamental frequency is of importance. For example, the simple non-linear function of voltage, $i = av - bv^3$, expressed earlier in this chapter, leads to:

$$\mathcal{I} = a\mathcal{V} - \tfrac{3}{4} b \mathcal{V}^3 \qquad (40)$$

(Cf. equation (31)). The method for deriving this kind of a function from instantaneous relationships has already been done for polynomials. It consists simply of expanding the function of the sinusoid into all of its frequency components by means of well-known trigonometric identities, and discarding all but the fundamental, as in equation (30). The procedure is therefore relatively simple in the power series case.

A procedure for determining the fundamental component of more general functions of sinusoids can also be derived. Again suppose $i = \psi(v)$. (Cf. equation (2).) If the new equation, analagous to (40), is expressed by $\mathcal{I} = F(\mathcal{V})$, then:

$$F(\mathcal{V}) = \frac{1}{\pi}\int_0^{2\pi} \psi(\mathcal{V}\cos\omega t)\cos\omega t \, d(\omega t) \qquad (41)$$

This equation has been derived by finding the fundamental Fourier component of $\psi(\mathcal{V} \cos \omega t)$ which is in phase with voltage, and thus in such a phase as to contribute power to the load. If there is any time lag between the output and the effect upon the active system produced by the output, there may be an out-of-phase component at the fundamental frequency. Such a component may be found from:

$$G(\mathcal{V}) = \frac{1}{\pi}\int_0^{2\pi} \psi(\mathcal{V} \cos \omega t) \sin \omega t \; d(\omega t) \tag{42}$$

The primary effect of the latter component is to shift the operating frequency; any effect upon the rate of build-up will be secondary.

The results obtained by using (41) may be compared with (40) by letting $\psi(v) = av - bv^3$, as before (cf. equation (2)). Then:

$$F(\mathcal{V}) = \frac{1}{\pi} \mathcal{V}\int_0^{2\pi} (a \cos^2 \omega t - b\mathcal{V}^2 \cos^4 \omega t) d(\omega t) \tag{43}$$

When the integral is evaluated:

$$F(\mathcal{V}) = a\mathcal{V} - \tfrac{3}{4} b\mathcal{V}^3 \tag{44}$$

(cf. equation (40)).

There are some disadvantages in using the expression of equation (2) for $\psi(v)$. Neither in a triode nor in a magnetron is there any effect which accompanies large r-f voltages which would lead to $\psi(v)$ being proportional to $(-v^3)$ for large r-f amplitude. Instead, the principal effect met with in a triode (or pentode or tetrode) is that either saturation or the low value of instantaneous plate voltage limits current on the positive swing of the grid, and current is cut off on the negative swing of the grid. Keeping in mind that when $\psi(v) = v^{2n}$, where n is any integer, $F(\mathcal{V}) = 0$ (cf. equation (41)), an odd function for $\psi(v)$ will be assumed such that:

$$\begin{aligned} i = \psi(v) &= kv, \quad \text{if } -A < kv < A \\ &= A \qquad\quad \text{if } kv > A \\ &= -A \qquad\;\; \text{if } kv < -A \end{aligned} \tag{45}$$

In a magnetron, the character of the non-linearity may be estimated by considering the electrons as being bunched in one region in respect to the r-f travelling wave, and completely absent elsewhere. It will be assumed that after a certain point, any greater intensity in the r-f electric field will not produce a greater r-f current by more effective bunching or greater total circulating charge. Thus, magnetron characteristics may also be investigated by using $\psi(v)$ as in (45).

A characteristic of the triode is that the current output is affected by the instantaneous plate voltage, as well as by grid voltage, and is expressed in terms of plate resistance. An analagous effect is met with in the magnetron. There is observed a reduction in electronic efficiency when loading is too light, apparently due to the fact that greater r-f amplitude causes electrons to strike the anode with greater radial velocity. It seems much more reasonable to associate either the plate resistance in a triode, or the lower efficiency of a magnetron, with the first power of r-f voltage, rather than with the third, as would happen if v were large in the expression $av - bv^3$. The first power assumption is also supported from experience. Therefore, the expression for $\psi(v)$ in (45) will be used in the following discussion, and the plate resistance, or the magnetron equivalent thereof, will be considered as being part of the equivalent passive circuit.

If (45) is substituted into (41), the integral may be easily evaluated by actually performing the computations for only one-quarter of a cycle. Then, if the value of k is greater than A:

$$F(v) = \frac{4}{\pi}\int_0^{\cos^{-1}\left(\frac{A}{kv}\right)} A\cos\omega t \; d(\omega t) + \frac{4}{\pi}\int_{\cos^{-1}\left(\frac{A}{kv}\right)}^{\frac{\pi}{2}} kv\cos^2\omega t \; d(\omega t)$$

$$= \frac{4}{\pi} A\sin\left(\cos^{-1}\frac{A}{kv}\right) + \frac{2}{\pi}kv\left[\frac{\pi}{2} - \cos^{-1}\frac{A}{kv}\right] - \frac{2}{\pi}kv\left[\frac{A}{kv}\sin\left(\cos^{-1}\frac{A}{kv}\right)\right] \qquad (46)$$

Equation (46) may be simplified by letting $\beta = \frac{\pi}{2} - \cos^{-1} \frac{A}{kV}$;

then $\frac{A}{kV} = \cos(\frac{\pi}{2} - \beta) = \sin\beta$, and $\sin(\cos^{-1}\frac{A}{kV}) =$

$\sin(\frac{\pi}{2} - \beta) = \cos\beta$. Therefore, (46) becomes:

$$F(V) = \frac{2}{\pi}(kV \sin\beta \cos\beta) + \frac{2}{\pi}\beta kV$$

$$= \frac{1}{\pi} kV (\sin 2\beta + 2\beta) \qquad (47)$$

where β is a function of V as described above. This expression is valid when kV is greater than $\underline{A}$. Whenever kV is less than $\underline{A}$, $F(V) = kV$.

When equations (25) and (35) are used in determining the current and voltage relationships in the passive circuit, the rate of build-up of oscillation may be found from the following expression:

$$V(2C\sigma + \frac{1}{R}) = \frac{kV}{\pi}(\sin 2\beta + 2\beta) \qquad (48)$$

Then, solving for σ :

$$\sigma = \frac{1}{2RC}\left(kR\,\frac{\sin 2\beta + 2\beta}{\pi} - 1\right) \qquad (49)$$

It is apparent that $\underline{k}$ has the dimensions of admittance, since in (45), kv is equal to a current.

It has already been stated that $\mathcal{V} = Ve^{\sigma t}$, where V is not variable with time. Then $\frac{d\mathcal{V}}{dt} = \sigma Ve^{\sigma t}$, or

$\sigma = \frac{1}{\mathcal{V}}\frac{d\mathcal{V}}{dt} = \frac{d}{dt} \ln \mathcal{V}$. Then (49) becomes:

$$d\mathcal{V} = \left\{\left[Rk \frac{2\beta + \sin 2\beta}{\pi}\right] - 1\right\} \frac{\mathcal{V}}{2} \frac{1}{RC}\, dt \tag{50}$$

Since RC has the dimensions of time, then (50) may be normalized, letting $\frac{t}{2RC} = \tau$. Then the following set of expressions results:

$$d\mathcal{V} = (Rk - 1)\mathcal{V}\, d\tau \qquad k\mathcal{V} < A \tag{51a}$$

$$d\mathcal{V} = \left(Rk \frac{2\beta + \sin 2\beta}{\pi} - 1\right) \mathcal{V}\, d\tau \qquad k\mathcal{V} > A \tag{51b}$$

Equations (51a) and (51b) may be integrated by numerical means. The resulting calculations are given in more detail in Appendix I. A typical solution is shown in Fig. 15. In this solution, the small-signal loop gain of the equivalent feed-back oscillator was taken as five, and A was taken as unity. To obtain the actual values of $\mathcal{V}/\mathcal{V}_{max}$ as shown in Fig. 15, the solution was finally normalized so that the steady-state value of $\mathcal{V}/\mathcal{V}_{max}$ was unity. The position of $\tau = 0$

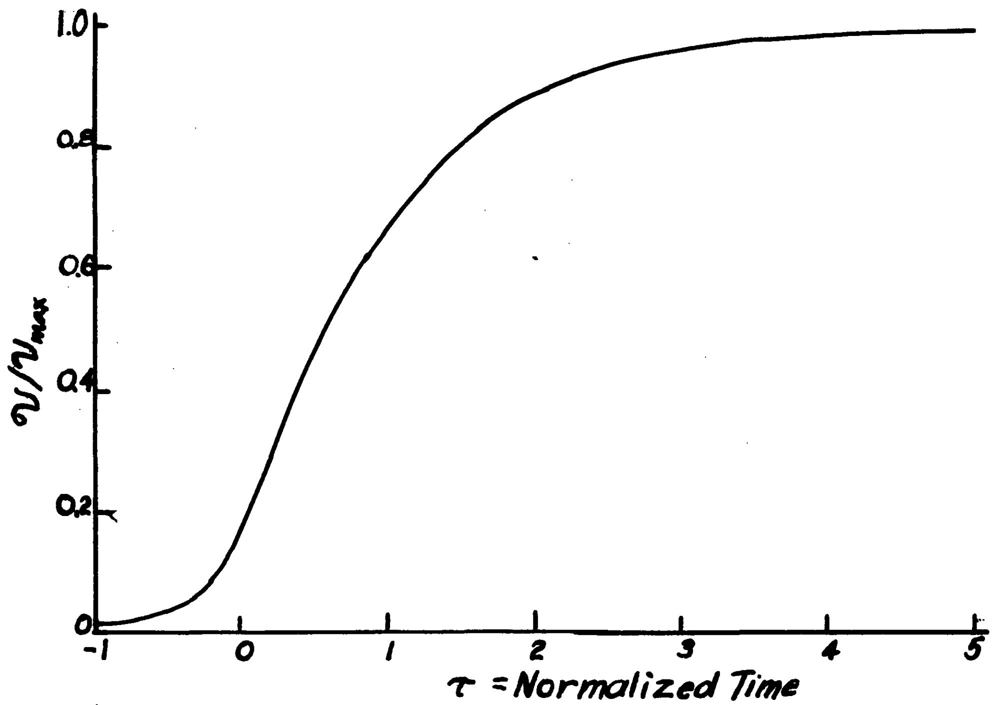

Fig. 15. Build-up of a feed-back oscillator with a-c voltage-current characteristics expressed by equation (45). The particular computation shown here corresponds to Rk = 5, where R and k are defined in the accompanying text.

was arbitrarily selected as corresponding to $k\mathcal{V} = A$, or in other words, to the boundary between (51a) and (51b). Therefore, the build-up to the left of $\tau = 0$ is exponential. To the right of $\tau = 0$, it gradually departs from the exponential form, and eventually approaches a maximum value as τ approaches infinity. This whole pattern is in agreement with the description of build-up of a magnetron made in Chapter III. Such agreement is to be expected, because fundamentally, the assumptions are similar. In Chapter III, it was assumed that the relationship between electric field and bunching was linear for small amplitudes, after which it approached a point where further increase in electric field leads to little or no increase in effectiveness in bunching.

5. Non-Linear Oscillators with More General Non-Linear Characteristics: Two Modes

The application of the principles of the preceding section to the two-mode problem is considerably more complicated. Although $\psi(v)$ may be expressed in the same way as before, the expression for current is much more difficult to arrive at. Current will be expressed here by $F_1(\mathcal{V}_1, \mathcal{V}_2) + F_2(\mathcal{V}_1, \mathcal{V}_2)$; F_1 corresponds to current at frequency ω_1, and F_2 corresponds to current at frequency ω_2. Now let $v = \mathcal{V}_1 \cos \omega_1 t + \mathcal{V}_2 \cos \omega_2 t$; furthermore, let ω_1 and ω_2 be integrally related, so that:

$$\mathcal{V}_1 \cos \omega_1 t + \mathcal{V}_2 \cos \omega_2 t = \mathcal{V}_1 \cos \omega_1 (t+T) + \mathcal{V}_2 \cos \omega_2 (t+T) \tag{52}$$

for all t, and in the period of time, T, both voltages execute an integral number of cycles.

To find the component of current which supplies power at frequency ω_1, the same procedure as that which led to (41) may be followed:

$$F_1(V_1, V_2) = \frac{1}{\pi n}\int_0^{2\pi n} \psi(v) \cos \omega_1 t \; d(\omega_1 t) \tag{53}$$

In (53), it has been assumed that the period T corresponded to n cycles of sin $\omega_1 t$. It is possible to rewrite (53) in the form:

$$F_1(V_1, V_2) = \frac{1}{\pi n}\sum_{p=0}^{n-1}\int_{2\pi p}^{2\pi(p+1)} \psi(V_1 \cos \omega_1 t + V_2 \cos \omega_2 t) \cos \omega_1 t \; d(\omega_1 t) \tag{54}$$

Each phase of $\omega_1 t$ is reached n times during the period, and for each phase there are n different phases of $\omega_2 t$. The totality of phases of $\omega_2 t$ for each phase position of $\omega_1 t$ leads to n equally spaced phase positions of $\omega_2 t$. Therefore:

$$F_1(V_1, V_2) = \frac{1}{\pi n}\sum_{p=0}^{n-1}\int_0^{2\pi} \psi\left[V_1 \cos \omega_1 t + V_2 \cos\left(\omega_2 t + \frac{2\pi p}{n}\right)\right] \cos \omega_1 t \; d(\omega_1 t) \tag{55}$$

As n becomes very large the summation may become an integral, and therefore, when the conditions of (52) are not met in any finite length of time, (55) may be replaced by:

$$F_1 = \frac{1}{\pi}\cdot\frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi}\Psi(V_1\cos\omega_1 t + V_2\cos\omega_2 t)\cos\omega_1 t \, d(\omega_1 t)\, d(\omega_2 t) \tag{56}$$

In equation (56), it is considered that time t, is not the independent variable in the double integration, but rather that $\omega_1 t$ and $\omega_2 t$ are independent variables. In using (56) to express conditions during build-up, it is necessary that value of $\omega_1 - \omega_2$ be much greater than either σ_1 or σ_2. It is necessary that the phase difference between the two components change rapidly as compared with the rate of build-up in order for the averaging effect implicit in the derivation to be reasonably well approximated. The effect which must be avoided is that which occurs when the two frequencies are nearly equal. When the voltages are nearly in phase, the output power over one cycle is nearly proportional to $(V_1 + V_2)^2$, rather than to $V_1^2 + V_2^2$; the excess power averages out over a large number of cycles. It is desirable that the time required for such averaging to be effective should be short in terms of the build-up time constant.

As a simple example of the application of equation (56), let $\psi(v) = av - bv^3$, as before. Then:

$$F_1(V_1, V_2) = \frac{a}{2\pi^2}\int_0^{2\pi}\int_0^{2\pi}(V_1 \cos \omega_1 t + V_2 \cos \omega_2 t) \cos \omega_1 t \; d(\omega_1 t) d(\omega_2 t)$$

$$- \frac{b}{2\pi^2}\int_0^{2\pi}\int_0^{2\pi}(V_1^3 \cos^3 \omega_1 t + 3V_1^2 V_2 \cos^2 \omega_1 t \cos \omega_2 t$$

$$+ 3V_1 V_2^2 \cos \omega_1 t \cos^2 \omega_2 t$$

$$+ V_2^3 \cos^3 \omega_2 t) \cos \omega_1 t \; d(\omega_1 t) d(\omega_2 t).$$

$$= aV_1 - \tfrac{3}{4} b V_1^3 - \tfrac{3}{2} b V_1 V_2^2 \tag{57}$$

This will lead to the same results as were derived in Section 3 of this chapter.

It is also of interest to examine mode interactions when the characteristics of the active circuit simply level off with large signal voltage, instead of decreasing in proportion to the cube of voltage as in (2). The characteristics expressed by (45) do not lead to an expression for $F_1(V_1, V_2)$ which is easily found from (56), because $\psi(v)$ is an implicit function in both frequency components. Thus when $\psi(v)$ changes from <u>kv</u> to <u>A</u> (cf. equation (45)), the values of $\omega_1 t$ and of $\omega_2 t$ at this point can be determined only by knowing the value of the function.

A reasonably good substitute for (45) is achieved by the following:

$$\psi(v) = a \tanh bv \tag{58}$$

This function has in common with (45) the properties of approaching (-1) for large negative values of v; of approaching (+1) for large positive values of v; and of having a slope of (+a) for small v. The integral in equation (56) would be easy to evaluate if $\psi(v)$ could be expanded in a Taylor series about v = 0; but such an expansion is valid only when $|bv| < (\pi/2)$. Therefore a numerical integration is again required. Integration of (56), using $\psi(v)$ as in (58), is carried out to a rough approximation in Appendix II. Values of $F_1(\mathcal{V}_1, \mathcal{V}_2)$ found this way are given in Table IV-1. Values of $F_2(\mathcal{V}_2, \mathcal{V}_1)$ may be found by reversing $\mathcal{V}_1$ and $\mathcal{V}_2$ in the table.

TABLE IV-1: Values of $F_1(\mathcal{V}_1, \mathcal{V}_2)$, to be multiplied by a.

$b\mathcal{V}_2$ \ $b\mathcal{V}_1$	0	0.5	1.0	1.5	2.0	3.0	5.0	∞
0	0	0.471	0.811	1.011	1.119	1.208	1.253	1.273
0.5	0	0.43	0.76		1.09			1.273
1.0	0	0.33	0.63		1.02			1.273
2.0	0	0.18	0.36		0.75			1.273
∞	0	0	0	0	0	0	0	

The rate of build-up of $\mathcal{V}_1$ may be determined from the following expressions:

$$2C_1\sigma_1 \mathcal{V}_1 = F_1(\mathcal{V}_1, \mathcal{V}_2) - \frac{\mathcal{V}_1}{R_1} \tag{59}$$

$$\sigma_1 = \frac{1}{2R_1C_1}\left[\frac{R_1}{\mathcal{V}_1} F_1(\mathcal{V}_1, \mathcal{V}_2) - 1\right] \tag{60}$$

The electronic conductances in the two modes may be expressed by $g_1(\mathcal{V}_1, \mathcal{V}_2)$ and $g_2(\mathcal{V}_2, \mathcal{V}_1)$, respectively, where $g_1 = F_1/\mathcal{V}_1$, and $g_2 = F_2/\mathcal{V}_2$. Values of g_1 as a function of $\mathcal{V}_1$, with $\mathcal{V}_2$ as a parameter, are plotted in Fig. 16. Then (60) may be rewritten:

$$\sigma_1 = \frac{1}{2C_1}\left(g_1 - \frac{1}{R_1}\right) \tag{61}$$

It may be shown in the same manner that:

$$\sigma_2 = \frac{1}{2C_2}\left(g_2 - \frac{1}{R_2}\right) \tag{62}$$

Using the results shown in Fig. 16, it is possible to make a plot of regions of build-up and decay of $\mathcal{V}_1$ and $\mathcal{V}_2$ in the same manner as Fig. 14. These results are shown in Fig. 17(a) for the case where $1/R_1 = 1/R_2 = 0.5a/b$. Thus,

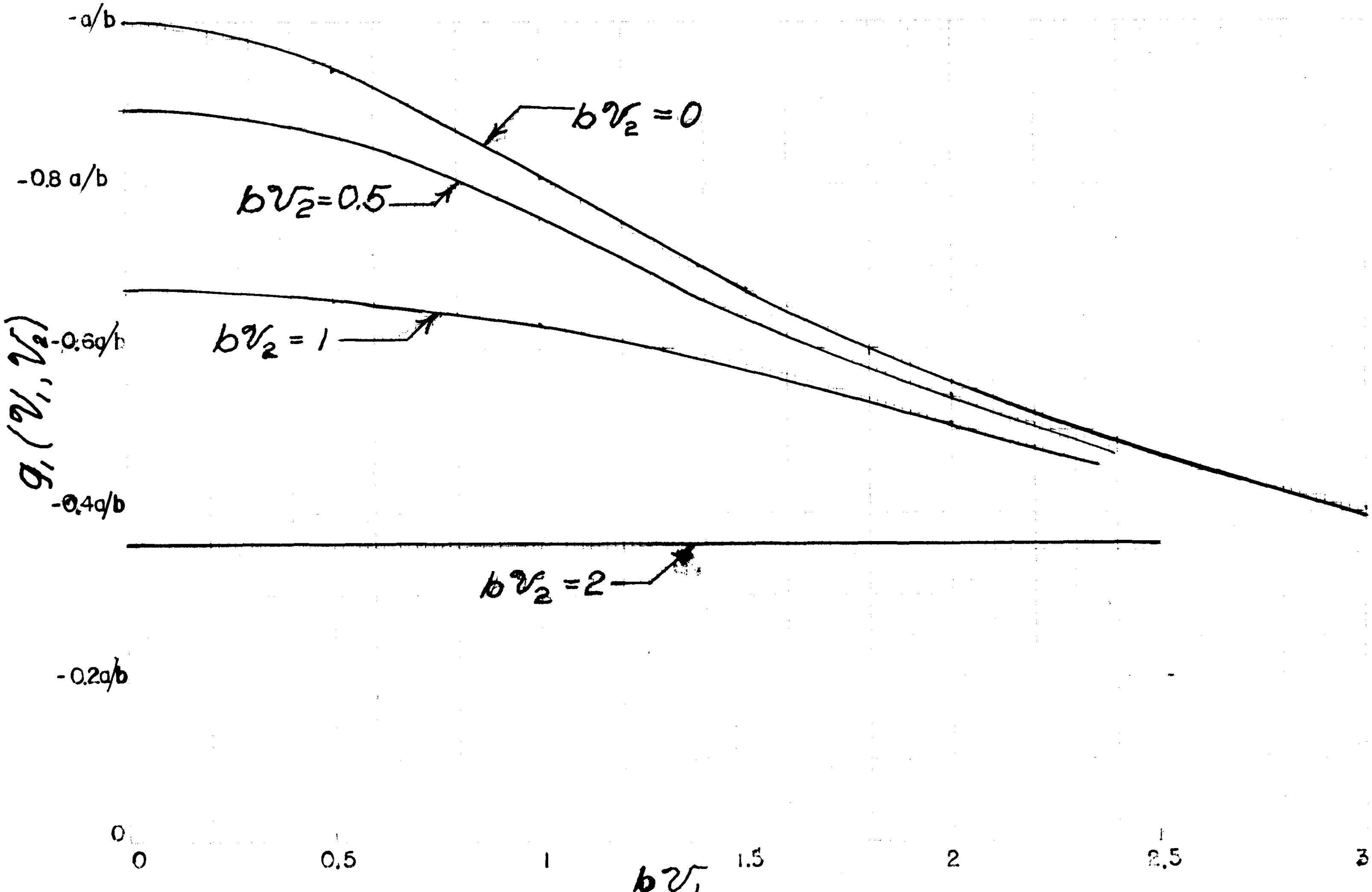

Figure 16. Electronic conductance, $g_1(\mathcal{V}_1, \mathcal{V}_2)$, of the non-linear generator, the characteristics of which are described by equations (56) and (58)--see text.

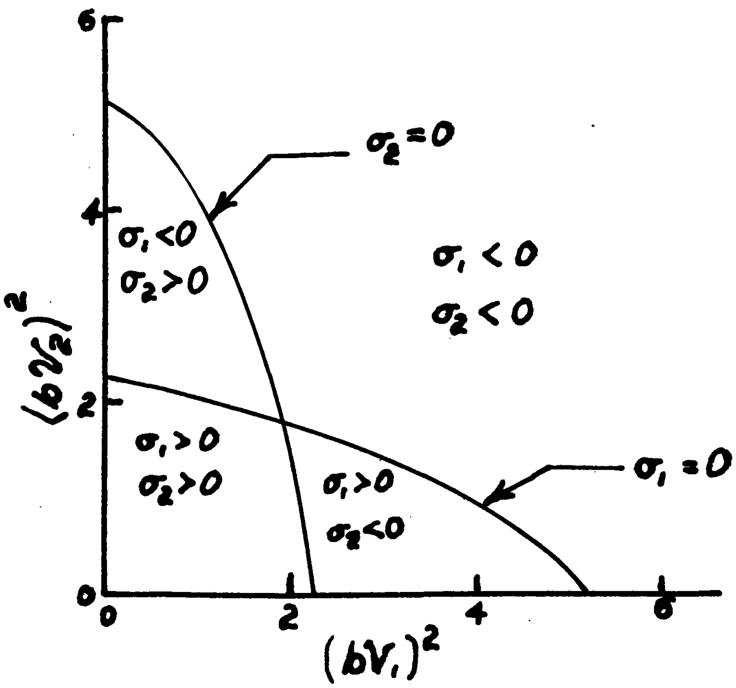

Figure 17a. Results presented in Fig. 16 are plotted here to show areas of build-up and decay of each mode. Either mode may oscillate stably: see text. (Cf. Fig. 14a.)

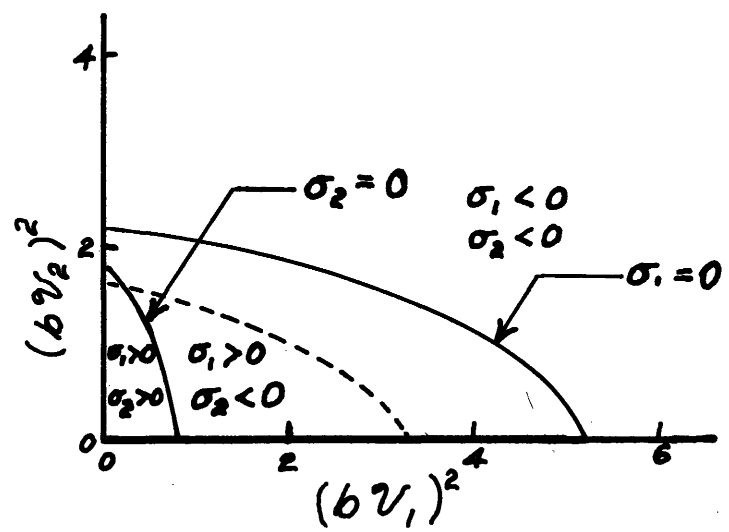

Figure 17b. Same as Figure 17a, except that the only stable oscillation possible is that for which $(b\mathcal{V}_1)^2 = 5.3$, if $\sigma_1 = 0$ is shown by solid line. Either mode may oscillate stably when $\sigma_1 = 0$ is shown by dashed line--see text. (Cf. Fig. 14b.)

Thus, the condition, $\sigma_1 = 0$ is satisfied for $b\nu_2 = 0$, $b\nu_1 = 2.3$; for $b\nu_2 = 0.5$, $b\nu_1 = 2.25$, etc. Here is exactly the same kind of results as in Fig. 14(a). For purposes of comparison, $(b\nu_1)^2$ and $(b\nu_2)^2$ are used as coordinates. Like the case illustrated by Fig. 14(a), stable oscillation in either mode is possible. Furthermore, the condition where $\sigma_1 = \sigma_2 = 0$, and both ν_1 and ν_2 are finite, is unstable, as in Fig. 14(a). There is no difference in principle in any respect; only the particular shapes of the $\sigma = 0$ curves are changed.

Another important aspect in which the solution expressed by equations (38) and (39) and the solution illustrated by Fig. 16 are similar is that in each case, the rate of build-up in one mode is affected less by the magnitude of its own oscillation than by the magnitude of oscillation in the other mode. For example, when $b\nu_1 = 0.5$, and $b\nu_2 = 0$, then this magnitude of ν_1 has reduced g_1 from 1.0(b/a) to 0.94(b/a); but when $b\nu_1 = 0$, and $b\nu_2 = 0.5$, then an equal magnitude of ν_2 has reduced g_1 to 0.89(b/a).

In Fig. 17(b), R_1 remains the same as in Fig. 17(a), but $1/R_2$ has been changed from 0.5a/b to 0.7a/b. This case is now, in principle, exactly the same as that illustrated in Fig. 14(b). Again, stable oscillation with ν_2 finite and $\nu_1 = 0$ is impossible.

It is significant in considering the kind of non-linear function expressed by (58) that the Taylor series expansion for (a tanh bv) which is valid for small values is of the form $\alpha v - bv^3 + \cdots$.[1] Therefore, close agreement between the solution derived using equation (58) and that derived from equation (2) is to be expected over a considerable range of values of v. This justifies van der Pol's assumption, expressed by equation (2), for the character of a non-linear current source, when the non-linearity is not very drastic.

6. Application of Non-linear Theory to Magnetrons

The most important result which comes from applying this kind of non-linear circuit theory to magnetrons is that large-amplitude oscillation in one mode has a strong tendency toward discouraging oscillation in other modes. It is particularly significant, as has been stated before, that the effect of the amplitude in one mode has more effect upon the rate of build-up of another mode than upon its own rate of build-up. The latter condition is necessary if the intersection of $\sigma_1 = 0$ and $\sigma_2 = 0$ in diagrams such as Fig. 14 and Fig. 17 (also cf. Fig. 8.47, reference No.1, Chapter 8, by F. F. Rieke, p. 383) is to be unstable. If this intersec-

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(1) According to Dwight, "Table of Integrals and Other Mathematical Data," (Macmillan, 1947), $\tanh x = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - - - -, \quad x^2 < \pi^2/4$

tion were stable, steady-state oscillation in the two modes simultaneously would be possible, and this is contrary to what is normally observed.

The results presented here also show that after one mode of oscillation has been established, it becomes much more difficult for a second mode to build up. This statement is in disagreement with all mode change theories in which conditions in the second mode, rather than in the originally oscillating mode, are considered to be the principal factors in a mode change. Instead, this kind of non-linear theory suggests that the most important factor in giving another mode a chance to build up is the weakening of the first mode. However, if the first one is not a very "strong" mode to start with, conditions may be not far removed from those in Fig. 17(b). Such a condition is illustrated by assuming that originally, σ_1 is represented by the broken line in Fig. 17(b), and that oscillation is taking place at frequency ω_2, with $\mathcal{V}_2$ finite and $\mathcal{V}_1 = 0$. Then let a slight change in conditions shift the line representing $\sigma_1 = 0$ from the position of the broken line to that of the solid line. Now $\mathcal{V}_1$ must build up, and $\mathcal{V}_2$ will be suppressed. It is under such circumstances, then, that a mode change can take place primarily as a result of conditions in the mode which builds up and suppresses the first one. However, the most significant fact brought out by the above discussion is that two modes have nearly equal strength, a condition comparable to that in Fig. 17(a) arises, and no small change associated with the non-oscillating mode will cause it to build up and suppress the oscillating mode.

CHAPTER V

MODE INTERACTIONS: STUDY OF ELECTRON MOTION

The study of one aspect of mode interactions will be taken up in this chapter from a more fundamental point of view than was used in the preceding chapter. Some drastically simplifying assumptions will be made concerning the nature of electron motion in the magnetron. Therefore, it is not practical to try to apply the results quantitatively to actual magnetrons. Nevertheless, it is possible to gain some useful qualitative information from this kind of a study.

For simplification, the cylindrical magnetron will be temporarily abandoned, and a linear magnetron, such as would result if the cylindrical magnetron were developed into an infinitely long structure with a cross section as shown in Fig. 18, and with a magnetic field perpendicular to the paper will be considered. The condition which will be assumed is that there is a large-amplitude steady-state r-f travelling wave present, and that the electron stream in that part of the interaction space under consideration here consists of tightly bunched electron spokes. A single electron in one of these spokes will be considered. This electron is assumed to be moving with a constant velocity in such phase as to contribute the maxi-

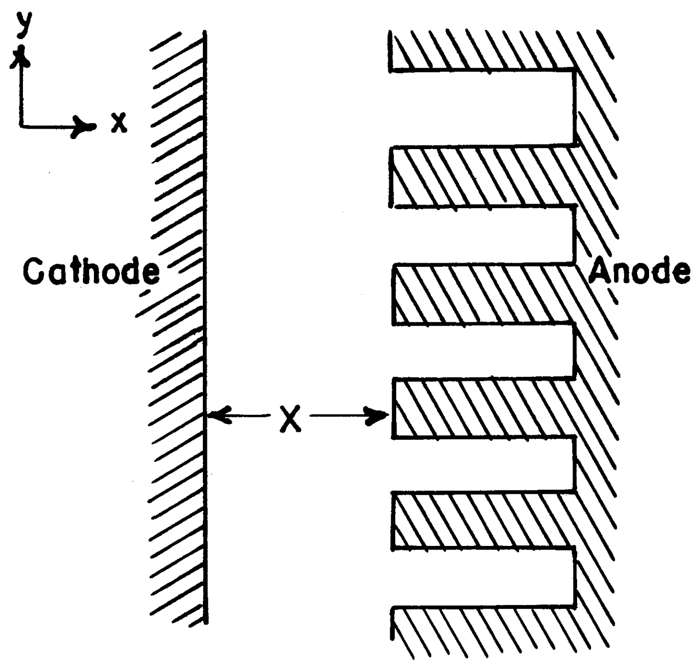

Figure 18. Cross section of linear magnetron.

mum possible energy to the above r-f travelling wave. The problem to be considered is the effect of this electron upon another r-f travelling wave, which will be of small amplitude.

If we assume that the effect of space charge upon the form of the r-f travelling wave is small, the resulting electric fields for the large amplitude travelling wave are described by:

$$E_x = \mathcal{E}_x \cosh k_x x \sin k_y(y - v_1 t) \tag{1}$$

$$E_y = \frac{k_x}{k_y}\mathcal{E}_x \sinh k_x x \cos k_y (y - v_1 t) \tag{2}$$

(Cf. reference no. 4, p. 11.) In (1) and (2), k_x and k_y are the propagation constants in the x- and y-directions, respectively, such that between $x = 0$ and $x = X$, $k_x^2 - k_y^2 = \omega^2/c^2$, where c is the free-space velocity of light; v_1 is the velocity of the travelling wave, and depends primarily on the cavity resonators, which appear as slots in Fig. 18.

The effect of the r-f magnetic field upon electron motion will be neglected in comparison with the externally applied constant magnetic field, B. Therefore, if a constant d-c field, E_o, between cathode and anode is included, the equations of electrons motion are:

$$\ddot{y} = \frac{e}{m}\left(E_y + B\dot{x}\right) \tag{3}$$

$$\ddot{x} = \frac{e}{m}\left(E_x + E_0 - B\dot{y}\right) \tag{4}$$

In (3) and (4), $\dot{x}$ and $\ddot{x}$ represent first and second derivatives, respectively, of x with respect to time, and $\dot{y}$ and $\ddot{y}$ represent similar derivatives of y. Here, e/m is the ratio of charge to mass of the electron. Signs have been chosen so that a positive acceleration corresponds to work done on the electron.

Since the large-amplitude travelling wave has a velocity equal to v_1, a new coordinate system may be chosen such that all motion may be expressed in relation to this travelling wave. Therefore, a new coordinate, u, may be introduced such that $u = y - v_1 t$. At the same time, the magnetic field may be expressed in terms of the cyclotron frequency(1) so that $\omega_c = eB/m$. Therefore, (3) and (4) may be replaced by:

$$\ddot{u} = \frac{e}{m}E_y + \omega_c \dot{x} \tag{5}$$

$$\ddot{x} = \frac{e}{m}\left(E_x + E_0\right) - \omega_c\left(\dot{u} + v_1\right) \tag{6}$$

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(1) Reference No.2, p.327.

The change in coordinates is also applied to (1) and (2), giving:

$$E_x = \mathcal{E}_x \cosh k_x x \sin k_y u \tag{7}$$

$$E_y = \frac{k_x}{k_y} \sinh k_x x \cos k_y u \tag{8}$$

If the electron under consideration travels in synchronism with the r-f travelling wave, its velocity in the y-direction is v_1, or in the new coordinates, its velocity is given by u = 0. If the electron is to give up the maximum possible energy to the tangential component of the r-f electric field (i.e., E_y), it is necessary either that $k_y u = (1+2n)\pi$, where n is zero or an integer, or for greater convenience, $\mathcal{E}_x$ may be considered to be negative, and the position of the electron may then be taken as u = 0.

In the analysis of electron motion which follows, electron motion in the y-direction will be considered to be relatively small, and therefore variations in electric field as a function of x will be neglected. Therefore, keeping in mind that $\mathcal{E}_x$ is negative, (7) and (8) become:

$$E_x = -A_x \sin k_y u \tag{9}$$

$$E_y = -A_y \cos k_y u \tag{10}$$

Here, A_x and A_y will be treated as positive and constant.

A kind of steady-state electron motion may now be described by setting $\ddot{x} = 0$, and $\ddot{y} = \ddot{u} = 0$. Then, (5) and (6) are reduced to:

$$\frac{e}{m} A_y \cos k_y u = \omega_c \dot{x} \tag{11}$$

$$\frac{e}{m} A_x \sin k_y u = \frac{e}{m} E_0 - \omega_c(\dot{u} + v_1) \tag{12}$$

For synchronism, $\dot{u} = 0$. If the electron is to contribute maximum energy to the tangential electric field, $u = 0$. These conditions change (11) and (12) to:

$$\frac{e}{m} A_y = \omega_c \dot{x} \tag{13}$$

$$0 = \frac{e}{m} E_0 - \omega_c v_1 \tag{14}$$

The requirement placed upon E_o by (14), in terms of ω_c and v_1, cannot, in practice, be met exactly. However, the picture is not changed greatly by small changes in E_o. The same kind of steady-state may be achieved by a small shift in the position of the electron. For small u, $\cos k_y u = 1$, and $\sin k_y u = k_y u$. Therefore, as a result of small changes in E_o, (13) is unchanged, and (14) may be replaced by:

$$\frac{e}{m} A_x k_y u_0 = \frac{e}{m} E_0 - \omega_c v_1 \tag{15}$$

The stability of the kind of steady state expressed by

(13) and (15) may be examined by means of the general solution to (5) and (6), taking $E_y = -A_y$, and $E_x = -A_x k_y u$, as above. Then equations (5) and (6) may be solved simultaneously to give:

$$\dddot{u} = \omega_c \frac{e}{m}\left(-A_x k_y u + E_0\right) - \omega_c^2(\dot{u} + v_1) \tag{16}$$

or, $$\dddot{u} + \omega_c^2 \dot{u} + Ku = \omega_c \frac{e}{m} E_0 - \omega_c^2 v_1 \tag{17}$$

A constant, **K**, has been introduced to replace $\omega_c(e/m)A_x k_y$.

Stability may be determined by setting the left-hand side of (17) equal to zero. The Laplace transform of the left-hand side of (17), neglecting initial conditions, is $s^3U + \omega_c^2 sU + KU$, where $\underline{U}$ is the transform of $\underline{u}$. Stability might be determined by solving for $\underline{s}$ in the following equation:

$$s^3 + \omega_c^2 s + K = 0 \tag{18}$$

Equation (18), and therefore equation (17), represents a stable system if none of the solutions for $\underline{s}$ in (18) have real parts which are positive. According to Gardner and Barnes,(1) an equation of the form of (18) may be examined for stability without actually solving, by Routh's method.

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(1) The application of Routh's method for determining stability is described by Gardner and Barnes, Transients in Linear Systems, (John Wiley and Sons, 1942), pp. 197-201.

First the coefficients are arranged as follows:

$$\begin{array}{c|cc} s^3 & 1 & \omega_c^2 \\ s^2 & 0 & K \end{array}$$

Normally, the first term in the next row (corresponding to s^1) would be formed with the first term in the second (s^2) row in the denominator. This is impossible here, since the latter term is zero. In order to allow the procedure to be carried further, a new equation will be written, substituting $\underline{p}$ for $1/s$:

$$Kp^3 + \omega_c^2 p^2 + 1 = 0 \tag{19}$$

Any roots of (18) in which the positive part of $\underline{s}$ is real will lead to roots of (19) in which the positive part of $\underline{p}$, or $1/s$, is real. Therefore, Routh's method can be applied to (19) to determine stability. In order to determine stability of a system associated with an equation of the same form as (19), for example, $b_3p^3 + b_2p^2 + b_1p + b_o = 0$, the coefficients are set down in the following manner:

$$\begin{array}{c|cc} p^3 & b_3 & b_1 \\ p^2 & b_2 & b_o \\ p^1 & \dfrac{b_2 b_1 - b_3 b_o}{b_2} & \\ p^0 & b_o & \end{array}$$

The number of roots for which the real part of p is positive is equal to the number of times the sign of the term in the first column changes, going from top to bottom. The above procedure may be applied to (19), and the coefficients set down in the same way as above:

$$\begin{array}{c|cc} p^3 & K & 0 \\ p^2 & \omega_c^2 & 1 \\ p^1 & -\frac{K}{\omega_c^2} & \\ p^0 & 1 & \end{array}$$

The number of roots for which the real part of p is positive is two, because there are two changes of sign in the first column. Therefore, there are two roots of (18) with real parts of s which are positive, and the system must be unstable.

As a result of the instability described here, small deviations of electrons from the position u = 0 continue to increase in oscillating fashion, and large deviations remain large. The deviations are limited by non-linearities which were left out when the approximations which led to (17) were made. Detailed orbital calculations, using self-consistent field methods to take space-change effects into account, indicate that the electron stream actually continues to lie near the region of maximum tangential electric field.(1)

(1) Reference No.1, Chapter 6 (by L.R.Walker),pp.274-282.

To investigate the interaction of the large-amplitude oscillations, described above, and an additional small-amplitude r-f travelling wave, a new wave will be introduced into the system. A pair of r-f field equations will be written, which are analagous to (9) and (10), to describe the new r-f wave:

$$E_x' = B_x \sin k_y'(y - v_2 t) \tag{20}$$

$$E_y' = B_y \cos k_y'(y - v_2 t) \tag{21}$$

When (9), (10), (20), and (21) are compared, A_x is much greater than B_x, and A_y is much greater than B_y. The simplifying assumptions involving (13) and (14), which led to the steady-state approximation to electron motion, will again be adopted, because orbital calculations show that, in principle, this is a correct view of electron motion, in spite of the instability which has been pointed out. This concept of electron motion will be subjected to perturbation by the r-f wave described by (20) and (21). Since it is the perturbations which will be of greatest interest, an inaccurate assumption as to the unperturbed electron motion will be less harmful than it might be if results were based more directly on that assumption. The equations of electron motion now become:

$$\ddot{u} = \frac{e}{m}\left[-A_y \cos k_y u + B_y \cos k_y'(u + v_3 t)\right] + \omega_c \dot{x} \tag{22}$$

$$\ddot{x} = \frac{e}{m}\left[-A_x \sin k_y u + B_x \sin k_y'(u + v_3 t)\right] - \omega_c \dot{u} \tag{23}$$

where $v_3 = v_1 - v_2$. When (22) and (23) are solved simultaneously:

$$\dddot{u} = \frac{e}{m}\left[k_y \dot{u} A_y \sin k_y u - k_y'(\dot{u} + v_3) B_y \sin k_y'(u + v_3 t)\right.$$
$$+ \omega_c \frac{e}{m}\left[-A_x \sin k_y u + B_x \sin k_y'(u + v_3 t)\right]$$
$$- \omega_c^2 \dot{u} \tag{24}$$

As before, it will be assumed that deviations of electrons from u = 0 are small. This assumption implies that $\sin k_y u = k_y u$, and $\cos k_y u = 1$, to a reasonable degree of approximation, as in (15). Furthermore, second and higher powers of u will be neglected, along with terms which include $\dot{u}u$, etc. Also, B_x and B_y are small, so that terms including products involving B_x or B_y in combination with u will be neglected. Other expressions which are found in

(24) are sin $k_y'(u + v_3t)$ and cos $k_y'(u + v_3t)$. These may be approximated in the following manner:

$$\begin{aligned}\sin k_y'(u+v_3 t) &= \sin k_y'u \cos k_y'v_3 t \\ &\quad + \cos k_y'u \sin k_y'v_3 t \\ &\approx k_y'u \cos k_y'v_3 t + \sin k_y'v_3 t\end{aligned} \tag{25}$$

$$\begin{aligned}\cos k_y'(u+v_3 t) &= \cos k_y'u \cos k_y'v_3 t \\ &\quad - \sin k_y'u \sin k_y'v_3 t \\ &\approx \cos k_y'v_3 t - k_y'u \sin k_y'v_3 t\end{aligned} \tag{26}$$

Using the above approximations, and letting $k_y'v_3 = h$, equation (24) may be rewritten:

$$\dddot{u} + \omega_c^2 \dot{u} = \frac{e}{m}\left[-hB_y \sin ht\right] + \omega_c \frac{e}{m}\left[-A_x k_y u + B_y \sin ht\right] \tag{27}$$

or,

$$\dddot{u} + \omega_c^2 \dot{u} + Ku = \frac{e}{m}\left(-hB_y + \omega_c B_x\right) \sin ht \tag{28}$$

In (28), K has been substituted for $\omega_c(e/m)A_xk_y$, as in (17). Therefore, the left-hand side of (28) will be recognized as being identical with the left-hand side of (17).

In practical magnetrons, ω_c is of the same order of magnitude as the operating frequency. It may be somewhat higher, but is rarely much less. The value of $k_y'v_2$ represents the angular frequency (i.e., $2\pi f$) of the small-amplitude mode of oscillation. Since it is necessary that v_1 and v_2 be not very different in order that the two modes may compete under any one set of conditions, $k_y'(v_1 - v_2)$, or h, may be expected to be much smaller than ω_c. Furthermore, when the velocity of propagation of the wave (i.e., v_2) is much less than that of a wave in free space, k_x' is nearly equal to k_y'.(1) The quantity B_x, equal to $\mathcal{E}_x' \cosh k_x'x$, is almost always larger than B_y, which is equal to $(k_x'/k_y') \times \mathcal{E}_x' \sinh k_x'x$, if k_x' is nearly equal to k_y'. For small values of $k_x'x$, B_x is much larger than B_y. Therefore, since ω_c is much greater than h, and B_x is greater than B_y, it must also be true that $\omega_c B_x$ is much greater than hB_y. Therefore, the quantity in equation (28) in parentheses, on the right-hand side of the equation, is positive, and a positive constant **D** will be introduced such that (28) becomes:

$$\dddot{u} + \omega_c^2 \dot{u} + K u = D \sin ht \tag{29}$$

Equation (29) will now be examined for a particular solution in the presence of the small-amplitude r-f wave. A solution of the form, $u = \alpha \sin ht + \beta \cos ht$, will be assumed, and substituted into (29). If the solution is to be

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(1) From the wave equation, $k_x'^2 - k_y'^2 = \frac{\omega'^2}{c^2}$. In a practical magnetron, $k_x' \gg \frac{\omega'}{c}$ and $k_y' \gg \frac{\omega'}{c}$.

valid for all values of time, it must be possible for the coefficients of <u>sin ht</u> and of <u>cos ht</u> to be equated independently. Such a procedure leads to the following equation, when coefficients of <u>sin ht</u> are equated:

$$h(h^2 - \omega_c^2)\beta + K\alpha = D \tag{30}$$

In a similar manner, coefficients of <u>cos ht</u> may be equated:

$$-h(h^2 - \omega_c^2)\alpha + K\beta = 0 \tag{31}$$

When (30) and (31) are solved simultaneously:

$$\alpha = \frac{KD}{h^2(\omega_c^2 - h^2)^2 + K^2} \tag{32}$$

$$\beta = \frac{-hD}{h^2(\omega_c^2 - h^2)^2 + K^2} \tag{33}$$

Since ω_c^2 is greater than h^2, as discussed before, and all of the other quantities involved are positive, then α is positive, while β depends on the sign of h. The work, W, done on the electron by the tangential electric field, is found by $\int eE_y dy$, where E is the entire electric field in the y-direction. Here, we are interested only in E_y', which is equal to $B_y \cos k_y'(u + v_3 t)$. The increment of distance is given by $dy = du + v_1 dt$, where $du = (h\alpha \cos ht - h\beta \sin ht)dt$. When $\int eE_y' dy$ is found for the whole cycle of ht, the total work done on the electron by the E_y' component of the electric field is:

$$W = e\int_0^{\frac{2\pi}{h}}\left[B_y \cos k_y'(u+v_3 t)\right]\left[h\alpha \cos ht - h\beta \sin ht + v_1\right] dt \tag{34}$$

In (26) it was shown that $\cos k_y'(u + v_3 t)$ is approximately equal to $\cos ht - k_y' u \sin ht$. Since $\underline{\alpha}$ and $\underline{\beta}$ are small terms, having been generated by a perturbation, any terms which would include the product of $\underline{u}$ with $\underline{\alpha}$ or $\underline{\beta}$ would be very small. Therefore, the only term in the product which leads to any net value of $\underline{W}$ is that containing $\underline{\cos^2 ht}$. The work done on the electron by the electric field during one cycle is therefore:

$$\begin{aligned} W &= e B_y \int_0^{\frac{2\pi}{h}} h\alpha \cos^2 ht \; dt \\ &= \pi e B_y \alpha \\ &= \pi e B_y \frac{KD}{h^2(\omega_c^2 - h^2) + K^2} \end{aligned} \tag{35}$$

The most important result here is that the perturbations of electron motion produced by the small-amplitude wave cause the electron to absorb energy from it. It is significant to observe that for small values of A_x (proportional to K by definition), the amount of energy absorbed is proportional to A_x, provided of course that bunching is more or less complete. For greater values of A_x, the rate of energy absorption by an electron may decrease as a result of an increase in A_x if the denominator, proportional to a constant plus K^2, increases proportionately more rapidly than the numerator, which is proportional to K.

As a consequence, the small-amplitude r-f travelling wave does not interact with the single electron, described here, in such a manner as to lead to regeneration, but instead, it must supply energy to the electron. Therefore, this discussion of mode interactions leads to a somewhat similar conclusion as was reached in Chapter IV. This conclusion is that <u>large-amplitude oscillation in one mode has a strong tendency toward discouraging oscillation in other modes</u>.

The analysis of electron motion carried out in this chapter suffers from some rather drastic approximations, which were necessary in order to reach any conclusions at all. However, there appears to be little reason to believe that the results obtained here are not correct in principle. The calculation here was carried out for one electron at one radial position. The integration led to an average effect for electrons at this radius, and a further summation of electrons at all radii should lead to an effect for the entire electron stream; this effect for the entire electron stream must remain the same in principle as for the electrons at one radius only.

CHAPTER VI

MODE STABILITY AND MODE INTERACTION EXPERIMENTS

In this chapter, experimental data will be discussed which pertain to various mode problems. An experiment will be described which is intended to measure as directly as possible some of the mode interaction phenomena described in Chapters IV and V. Other experiments which will also be discussed provide information which supports the preceding theory in terms of both mode stability and fundamental causes of mode changes.

1. Mode Interactions

The effect of large-amplitude oscillation in one mode upon resonances corresponding to other modes has been discussed theoretically from two different points of view in Chapters IV and V. The experiment described here was carried out in order to measure this kind of an effect as directly as possible. The object was to determine the loaded Q of a resonance corresponding to one mode when the magnetron was actually oscillating in another mode.

A block diagram of equipment used in this experiment is shown in Fig. 19. A more detailed description of the equipment is included in Appendix III. The magnetron (718EY) oscillated in the π-mode (4/4/8), and the characteristics of the $n = 3$ resonance were measured by conventional "cold-

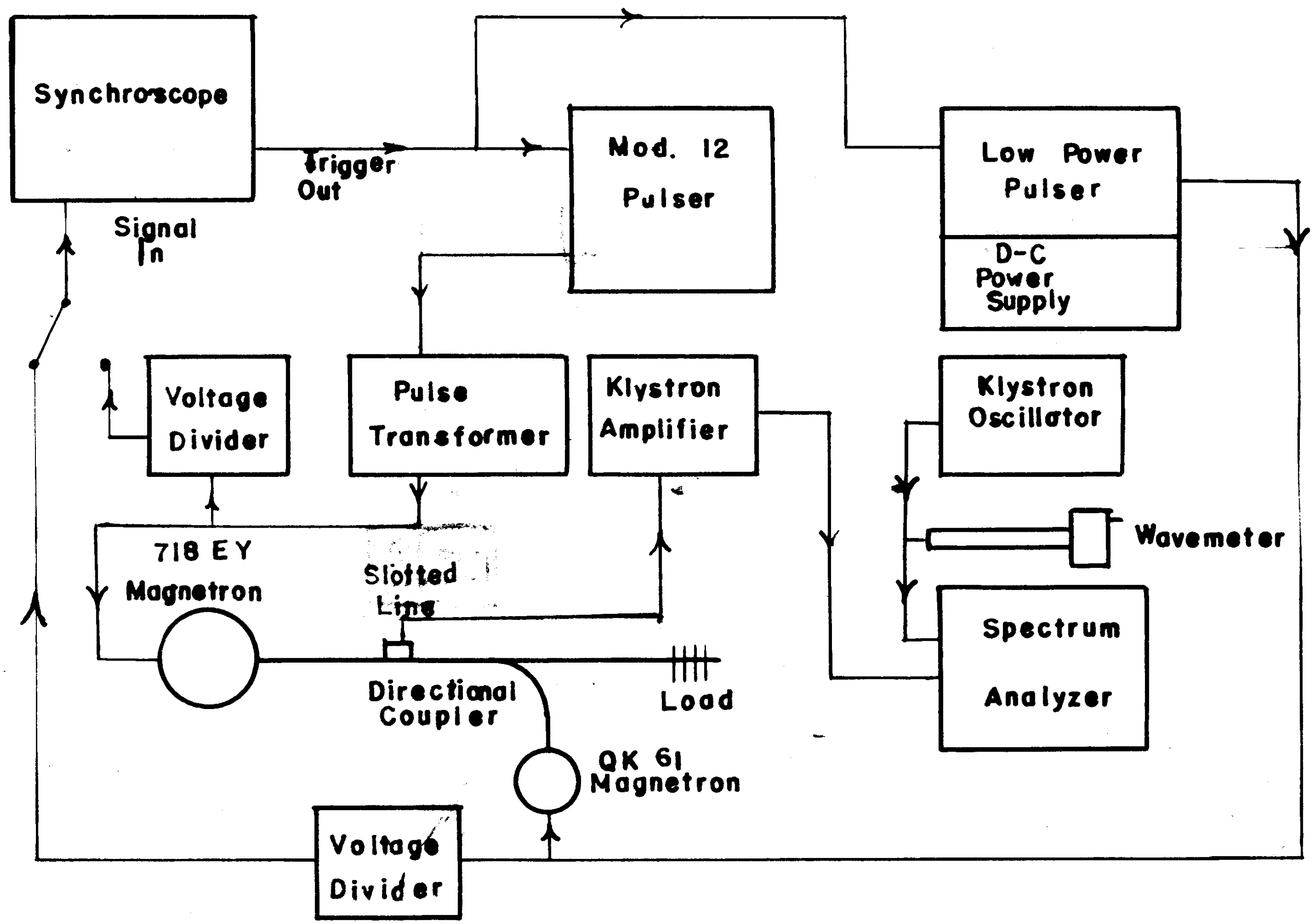

Figure 19. Block diagram of equipment used in mode interaction test.

test" methods,[1] based on standing-wave measurements.

A signal, which would be tuned through a wide enough range to study the n = 3 resonance under a wide variety of conditions, was supplied by the QK-61 magnetron through a directional coupler. Standing-wave measurements were made by means of a slotted line, which picked up a signal which was fed to the spectrum analyzer through the klystron amplifier. The klystron and the spectrum analyzer were both tuned to the frequency of the QK-61, thus eliminating the π-mode signal so completely that it could not be observed on the spectrum analyzer. Thus it was possible to measure characteristics of the n = 3 resonance while the π-mode was oscillating.

Since the 718EY magnetron was oscillating only during the applied 5-microsecond pulse, the standing wave measurements were made only within this period. The signal supplied by the QK-61 was therefore pulsed with a duration of 3.5 microseconds, and the latter pulse was synchronized so that it began after the 5-microsecond pulse had begun, and ended before the 5-microsecond pulse ended. It was also found desirable to pulse the klystron together with the QK-61, because it was observed that the 718EY output contained considerable energy at the n = 3 frequency during the build-up transient and at the end of the pulse. Therefore, the only

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(1) Reference No.2, pp.89-95.

signal reaching the spectrum analyzer was that which occurred during the interval in which the small-amplitude signal was supplied to the system.

The first series of tests of this kind was performed with constant loading of the π-mode (i.e., magnetron coupled to r-f line according to specifications, and with the line terminated by a matched load). Magnetic field was held constant (1220 gauss), and four different values of input current were used. The resulting four sets of measurements of standing-wave ratio as a function of wavelength are plotted in Fig. 20. Oscillation was taking place in the π-mode only for the two highest values of anode current. In the first test, there was no power supplied to the 718EY, and in the second test, with low anode current, no coherent oscillation could be observed.

From the data in Fig. 20, the loaded Q of the n = 3 resonance was computed using the method described by Slater.[1] It should be noted that only in the first, and narrowest, curve, did the position of the standing-wave minimum shift by half of a wavelength as the wavelength passed through resonance. The results of these calculations are given in Table VI-1. Although the external Q is normally only a measure of coupling between the resonant system and the load, considerable variation is shown here, in spite of the fact

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(1) Reference No.2, pp. 89-95.

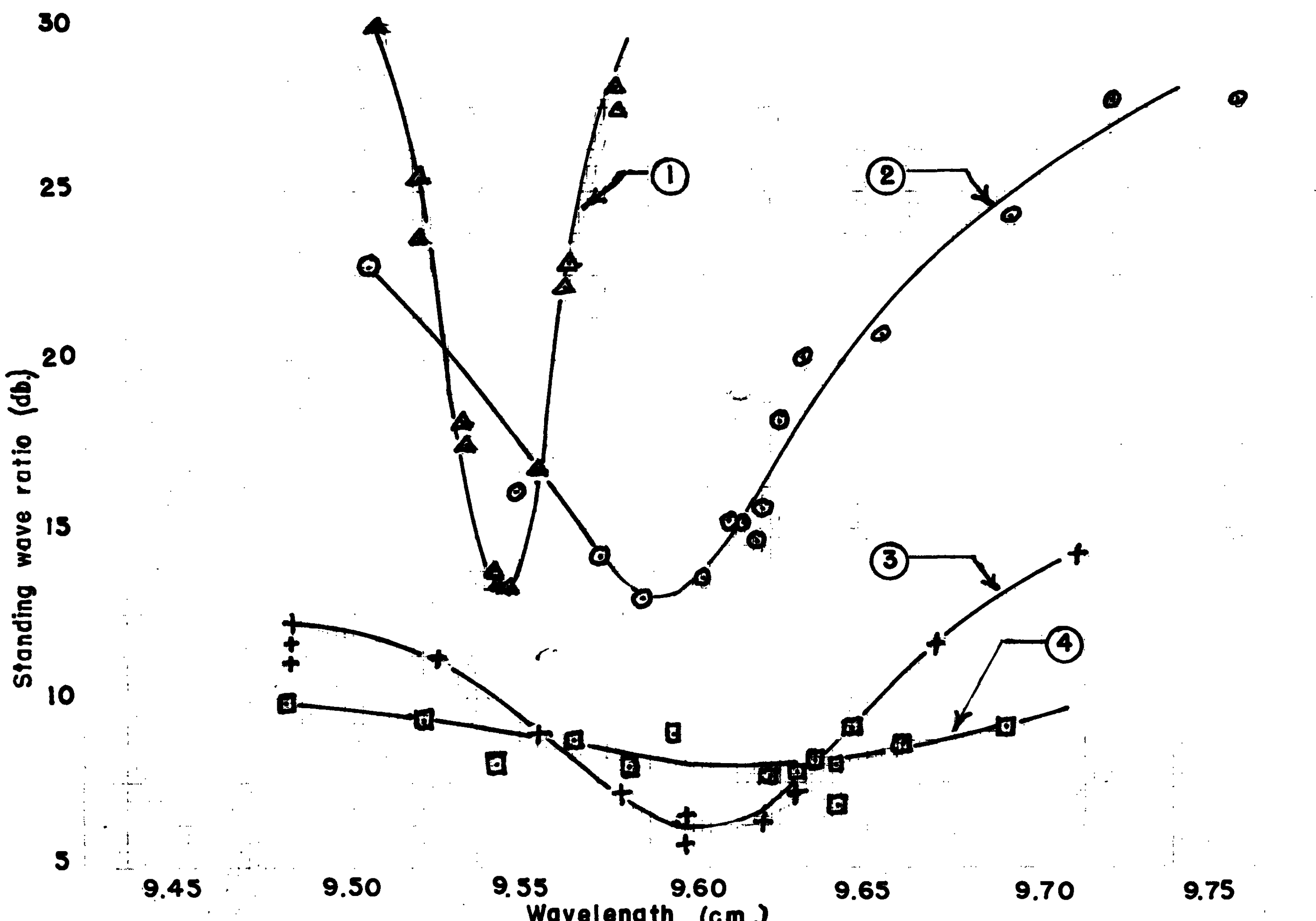

Figure 20. Stranding-wave measurements of 718EY magnetron: n = 3 mode of resonance. (1) No anode power. (2) Peak anode **current** = 1.1 amp., no oscillation. (3) Peak anode current = 6.8 amp., π-mode oscillation taking place. (4) Peak anode current = 9.9 amp., π-mode oscillation taking place. Magnetic field = 1220 gauss for each case.

that no change in the coupling was made. The change in the loading of the n = 3 resonance, according to theory, should appear only as a change in the "unloaded" Q, as determined from external measurements.

TABLE VI-1

Measurements of n = 3 Resonance in 718EY Magnetron

Peak Anode Current	External Q	Unloaded Q ("internal"Q)	Loaded Q
0	344	1640	285
1.1	600	112	95
6.8(a)	420	108	86
9.9(a)	420(b)	45	40

(a) π-mode oscillation taking place

(b) External Q could not be obtained from graph. It was assumed to be the same as for the preceding value of anode current.

These results show that as intensity of oscillation increases, and anode current is also increasing, loading of the n = 3 resonance by the electron stream increases. Although the calculations for Q when peak anode current is 1.1 amperes appear to lead to a loaded Q nearly equal to that found from the calculations for 6.8 amperes, nevertheless, the 1.1-ampere curve in Fig. 20 appears narrower, leading to the appearance of much higher Q.

However, these results show definitely that the non-oscillating mode is loaded by the electron stream, and that for larger values of anode current, corresponding to stronger oscillation, the effect is greater. However, it is apparent in the 1.1-ampere case that such a loading effect is present, even when coherent π-mode oscillation is not. Therefore, these data are insufficient to evaluate the effect of the actual π-mode oscillation upon loading the non-oscillating mode, apart from effects resulting merely from the flow of anode current.

In Fig. 21, data are plotted for a similar set of tests (cf. Fig. 20), using another 718EY magnetron. Calculated results are given in Table VI-2.

TABLE VI-2

Measurements of n = 3 Resonance in 718EY Magnetron

Peak Anode Current (amp.)	External Q	Unloaded Q ("internal"Q)	Loaded Q
0	506	1680	388
1.1	600	87	76
7.9(a)	325	55	47
16	600	158	128

(a) π-mode oscillation taking place.

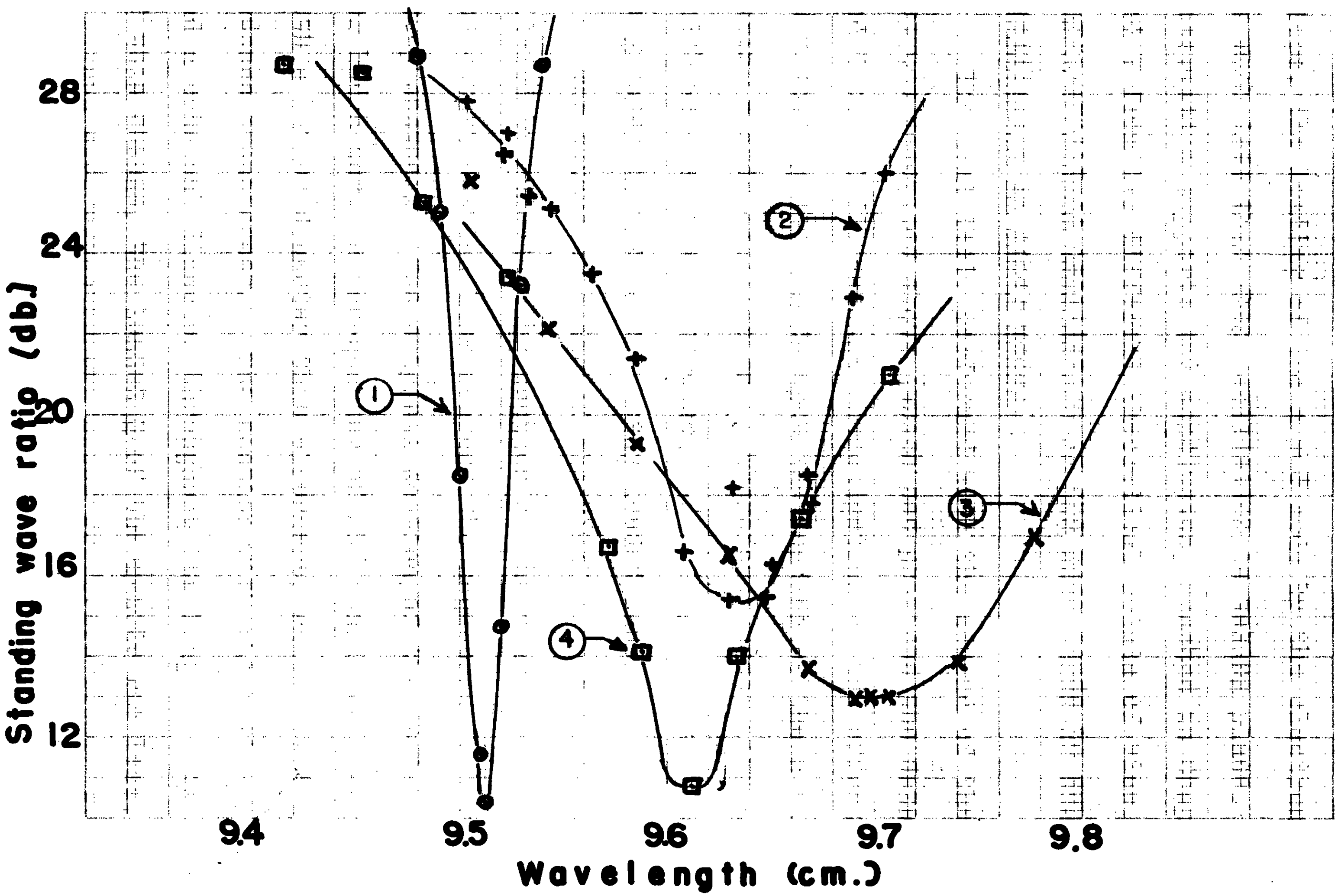

Figure 21. Standing-wave measurements of 718EY magnetron: n = 3 mode of resonance. (1) No anode power. (2) Peak anode current = 1.1 amp., no oscillation. (3) Peak anode current = 7.9 amp., π-mode oscillation taking place. (4) Peak anode current = 16 amp., no oscillation. Magnetic field = 1220 gauss for each case.

It can be seen both from Fig. 21 and from Table VI-2 that when the applied voltage is so high that oscillation stops, the loading of the $n = 3$ mode by the electron stream is much less than when oscillation is taking place. This observation supports the conclusion, reached in Chapters IV and V from two different theoretical points of view, that <u>large-amplitude oscillation in one mode tends to suppress oscillation in other modes</u>.

The next test, performed with the second of the two magnetrons described above (which had provided the data in Fig.21 and Table VI-2), was the investigation of the effect produced by unloading the π-mode upon the loading of the $n = 3$ mode, during π-mode oscillation. Therefore, under circumstances which were similar otherwise, r-f voltage in the π-mode should be greater in the more lightly loaded case.

The output coupling of the 718EY magnetron is easily altered by removing the section of the center conductor of the output coaxial line nearest the magnetron, and replacing it by another. The original section of the center conductor had a portion one-quarter of the π-mode wavelength long with a larger diameter than elsewhere, which acted as an impedance transformer. The section with which it was replaced had a constant diameter throughout its length. As a result of this change, the loaded $\underline{Q}$ of the π-mode was less.

The data found in the tests described above, for the two different conditions of r-f loading, are plotted in Fig.22, and calculated results are given in Table VI-3.

TABLE VI-3

Measurements of n = 3 Resonance in 718EY Magnetron: π-Mode Loading is Varied

	π-Mode Normally Loaded	π-Mode Lightly Loaded
1. Cold test of π-mode		
External Q	195	750
Unloaded Q	700	1200
Loaded Q	152	460
2. Cold test of n = 3 mode		
External Q	506	341
Unloaded Q	1680	1240
Loaded Q	388	258
3. "Cold test" of n = 3 mode during π-mode operation		
External Q	325	345
Unloaded (internal) Q	55	62
Loaded Q	47	53
4. R-f voltage developed in π-mode	$832/\sqrt{\omega C}$	$1760/\sqrt{\omega C}$

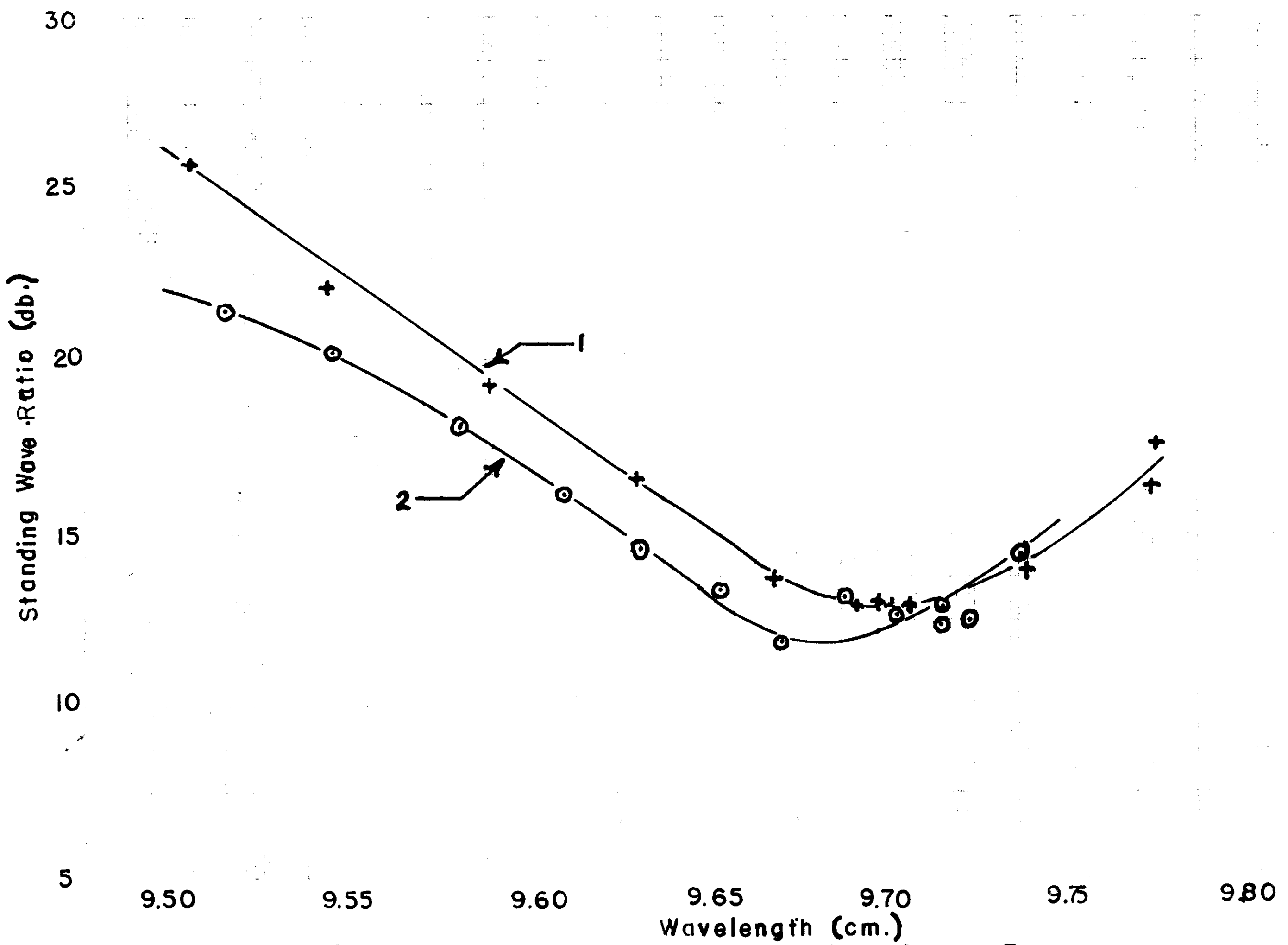

Figure 22. Standing-wave measurements of n = 3 resonance in 718-EY magnetron, with π-mode oscillations taking place. (1) Normal output coupling. (2) Output coupling altered to lighten π-mode loading.

When the π-mode is normally loaded, the loaded Q of the n=3 mode changes from 388, with no power applied, to 47, when 7 kv. is applied, with the peak anode current at 7.9 amperes, and π-mode oscillation taking place. When the π-mode is lightly loaded, the loaded Q of the n = 3 mode changes from 258, with no power applied, to 53, when 7 kv. is applied, with peak anode current 7.4 amperes, and the π-mode r-f voltage more than twice as much as in the normally loaded case. Therefore, <u>the higher value of r-f voltage has less effect upon the other mode than has the lower value of r-f voltage</u>. This observation is inconsistent with the results obtained from the non-linear oscillator theory in Chapter IV, in which greater r-f voltage in one mode should always have greater adverse effect on other modes. However, this observation is not inconsistent with the results of the electron-motion analysis of Chapter V, in which the adverse effect of one mode upon other modes may decrease with increasing r-f voltage, when the latter voltage becomes large enough.

It is also interesting to compare electronic conductances as a function of the magnitude of r-f voltage in the π-mode. In all of the results in Table IV-4, below, the anode voltage is 7.0 kv. and the magnetic field is 1220 gauss, as before. The electronic conductance in the π-mode for no r-f voltage was estimated from the shape of the detected r-f envelope observed during the early part of build-up. Means of calculating electronic conductance are described by Rieke.(1)

TABLE VI-4

Electronic Conductances in an Oscillating Magnetron

π-Mode Voltage	π-Mode Electronic Conductance	n=3 Electronic Conductance
	$\frac{G_{e,4}}{\omega_4 C_4}$	$\frac{G_{e,3}}{\omega_3 C_3}$
0	-0.013	0 (?)
$832/\sqrt{\omega_4 C_4}$	-0.0066	+0.019
$1760/\sqrt{\omega_4 C_4}$	-0.0022	+0.015

In the above table, negative conductance is regarded as supplying power, and positive as absorbing power. The quantities ω_4 and C_4 represent angular frequency and equivalent capacity, respectively, for the π-mode (n = 4), and ω_3 and C_3 represent similar quantities for the n = 3 mode. It is of interest to observe here that the positive electronic conductance in the n = 3 mode is, in the cases for which it was measured, much larger than the magnitude of negative

(1) Reference No.1, Chapters 7 and 8 (by F.F.Rieke).

conductance in the π-mode, except for very small π-mode r-f voltages.

The results of these tests are, to some extent, doubtful as to accuracy, because of the fact that it was not possible to plot very accurate curves of standing-wave ratio. Apparently the observed values of standing-wave ratio were subject to considerable random error, and the calculated values for loaded Q, based on the same resonance curve, showed considerable inconsistency. The variations of external Q were particularly disconcerting. When electronic conditions were changed, the loading of a resonant mode should be expected to occur entirely within the magnetron. Yet, there appeared to be substantial variations of external Q, even though no change had been made in the r-f output circuit. These effects are thought to be an experimental error not of a fundamental nature. Because the worst inconsistencies in external Q appear where the internal (or unloaded) Q has a much lower value, it is the internal Q which affects the loaded Q most under these circumstances, and inconsistencies in external Q are considered to be much less important than values of internal Q.

In spite of the apparent inconsistencies in data, the effects observed are so large that it is difficult to conceive how any reasonably large errors in measurements could obscure the correct results in principle.

2. Observations of Mode Changes

a. Mode Changes: Low-Power Rising-Sun Magnetron

The first experiment to be described here involves a specially constructed 18-vane low-power magnetron of the rising-sun type. Its dimensions and construction are described in Appendix IV. This magnetron was originally intended for c-w operation, but this was prevented because of the fact that under c-w conditions, cathode heating was excessive. Instead, operation with long pulses was carried on, with pulse durations of ten to twenty microseconds, or more.

All of the tests described here were performed with a magnetic field of 1410 gauss. With the r-f load matched to the output coaxial transmission line, the voltage-current relationships were as shown in Fig. 23. Under these conditions, three different modes were observed, labeled A, B, and C. These modes are described in Table VI-5. The identification of each mode was made by means of a rotating probe. The values of n, for modes B and C especially, were not perfectly clear, but the values specified in the table seem to be consistent with mode-spectrum theory for rising-sun magnetrons.(1) If values of *n* are known, the values of *γ* are quite unambiguous, because the starting voltages, func-

(1) Reference No.3, p.229.

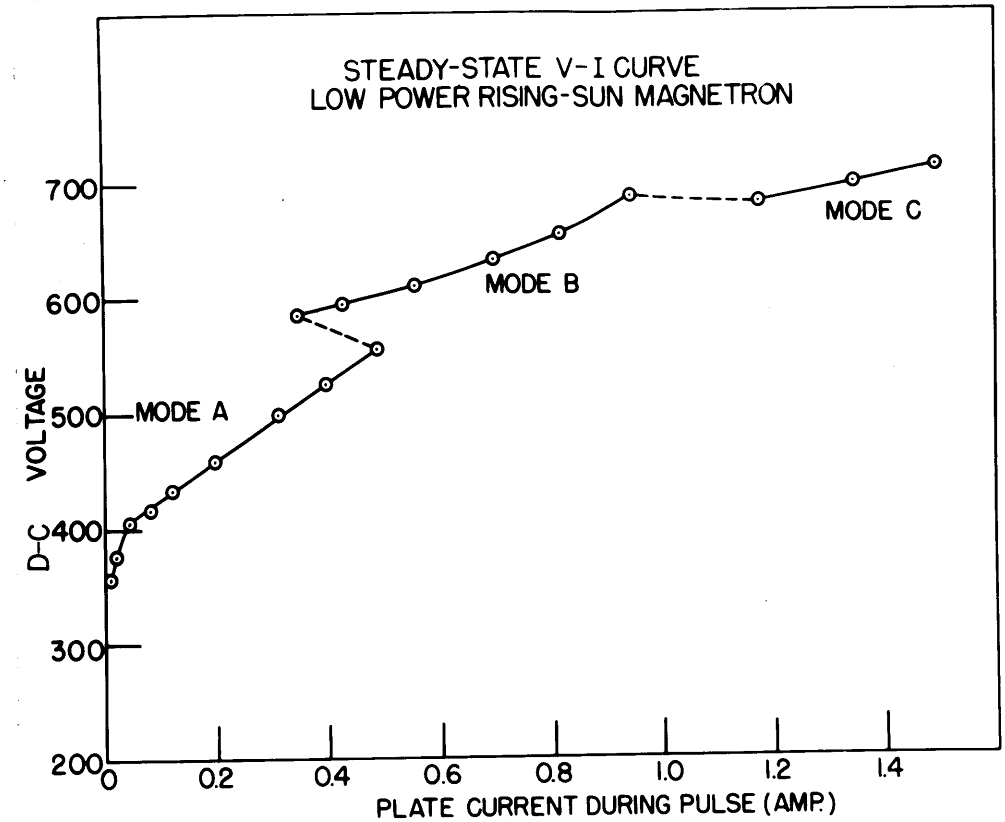

Figure 23. For identification of modes, see Table VI-5.
B= 1410 gauss.

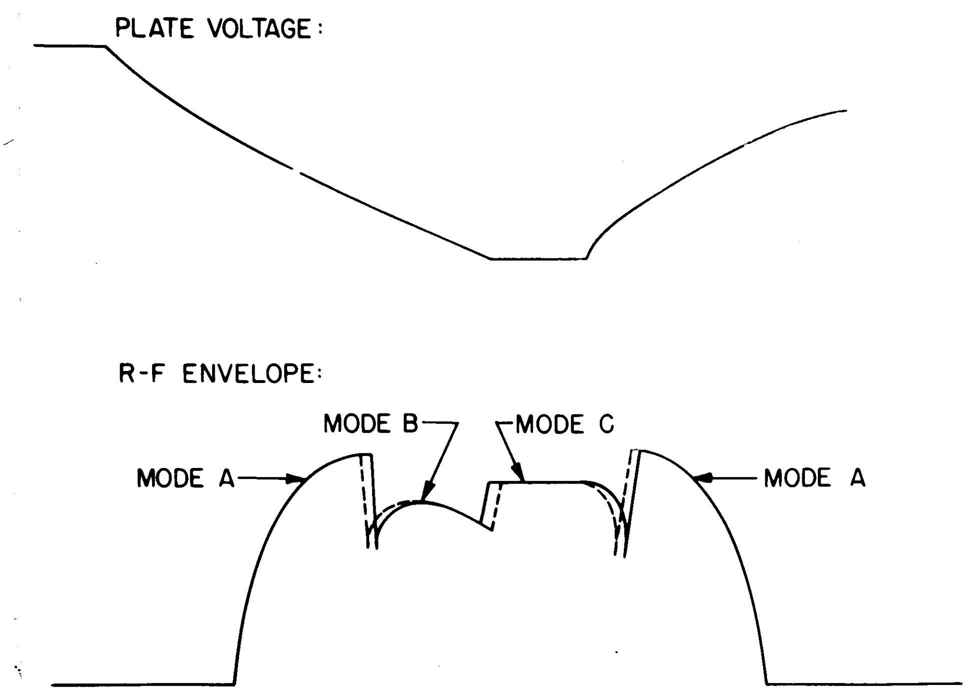

Figure 24. Drawings taken from synchroscope traces, showing how the positions of mode shifts are affected by changes in loading. For details, see text.

tions of $\underline{\gamma}$ and frequency, were observed.

TABLE VI-5

Mode Identification in Low-Power Rising-Sun Magnetron

	Identification of Mode	Wavelength (cm)
Mode A	9/9/18 (π-mode)	11.48
Mode B	6/3/18	14.25
Mode C	7/2/18	14.62

When a slowly rising voltage pulse (rise time, 20 microseconds) was applied to this magnetron, oscillation was observed successively in modes A, B, and C, and during the fall of voltage, oscillation shifted from C back to A; these changes are shown in Fig. 24.

The test described here was for the purpose of determining whether the change from one mode to the next depended primarily upon conditions in the original mode, conditions in the next mode, or a combination of both. To reach any satisfactory conclusions on this question, it was necessary to alter conditions in one mode without affecting others. The condition which was altered for each mode was the external loading. This was accomplished separately for each mode by means of an absorption-type wavementer, as shown in Fig. 25. The loading for each mode is changed by a small but appreciable amount as the wavemeter is tuned to that mode;

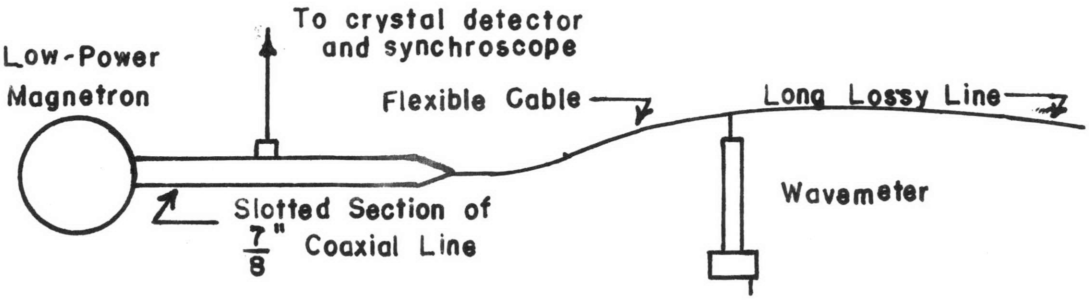

Figure 25. R-f portion of test equipment for load-variation experiment with low-power rising-sun magnetron.

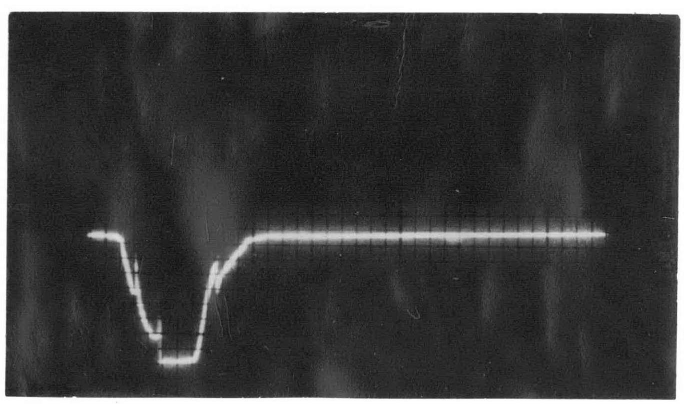

Figure 26. Current pulse associated with mode changes in low-power rising-sun magnetron. Conditions are the same here as in Figs. 23 and 24.

but tuning to the frequency of one particular mode has virtually no effect on other modes.

When the wavemeter was tuned to mode A, the boundary between A and B was changed from the position shown by the solid line in Fig. 24 to that shown by the dashed line. No effect, however, upon the boundary between C and A was produced thereby. Tuning the wavemeter to the frequency of mode B affected the ending of that mode in a manner shown by the dashed line, but did not affect its starting. Likewise, tuning the wavemeter to the frequency of mode C affected its ending, as shown by the dashed line, but did not affect its starting. These results indicate that the conditions for shifting from any one of these modes to another are determined primarily by the characteristics of the mode initially oscillating, and are not affected by the mode which is found immediately after the initial mode has become unstable, no matter whether the voltage shift which gave rise to the mode change is upward or downward.

These results are confirmed by synchroscope observation of the anode current, as shown in Fig. 26. Each of the mode change boundaries is accompanied by a vertical spike, indicating a sharp reduction of current during the transition. Since current in an oscillating magnetron can flow only as a consequence of oscillation, the reduction in current evidently corresponds to cessation of oscillation in the original mode, followed by the building-up of oscillation in the subsequent mode. It should be pointed out that the signal on the synchroscope (Fig. 26) has passed through a video amplifier, and therefore, the limited frequency response (about 10 mc.) may have reduced the sharpness of the spike which corresponds to the reduction of current.

b. Mode Changes: 2J54 Magnetron

The 2J54 magnetron has an eight-cavity strapped anode with hole-and-slot construction, fundamentally similar to the 2J32.(1) It differs from the 2J32 slightly in wavelength, and also because it is tunable. The characteristic which is of interest here is that it sometimes oscillates in the 5/3/8 mode at a slightly lower anode voltage than that for which it begins to oscillate in the π-mode. For certain values of magnetic field and a reflectionless output line, oscillation was observed to begin in the 5/3/8 mode, and to persist even after the applied voltage had reached a value at which the π-mode could build up. The r-f envelope observed under these circumstances is shown in Fig. 27. The step observed in the leading edge was found to correspond to the $n = 3$ resonance, and the value of applied voltage was identified as the 5/3/8 mode.

To determine the nature of the transition from the

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(1) Reference No.1, pp.751-756.

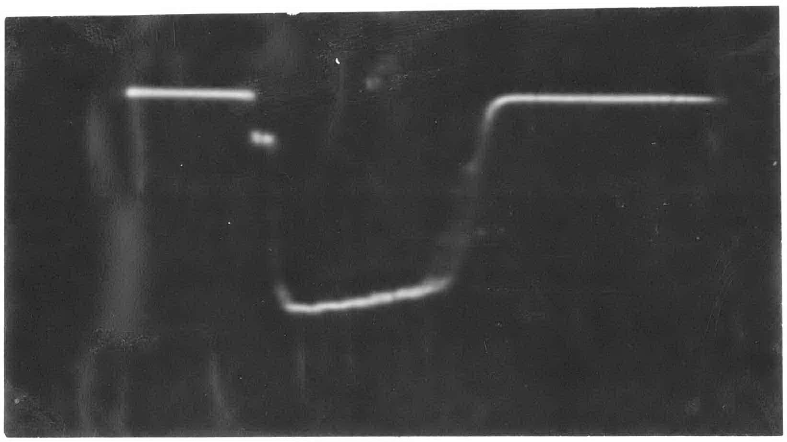

Fig. 27. R-f envelope for 2J54 magnetron, showing initial oscillation in the 5/3/8 mode, followed by π-mode oscillation. Pulse duration = 2 microseconds.

5/3/8 mode to the π-mode, the output coaxial line was terminated in a movable mismatch. The mismatch consisted of a movable short-circuit, with a high-power attenuator (about 10 db.) between the short circuit and the magnetron. Thus, the wave reflected by the short circuit represents about

0.01 in power and 0.1 in voltage of the wave travelling toward the load, when this measurement is made between the magnetron and the attenuator. Thus, a considerable change in magnetron loading could be made by the position of the reflection.

It was observed that as the short circuit was shifted in position, the time of the transition between modes also changed in relation to the start of the pulse. The change in the relative time of the transition could be made to be a relatively large part of the pulse duration if the value of applied voltage was only slightly greater than that at which the transition takes place. A shift of the position of the short circuit was made over a great enough distance so that the relative position of the mode transition moved from one extreme to the other and back again. The distance over which the short circuit had to be shifted, to accomplish the change from one extreme and back to the same extreme, was measured several times; this measurement was repeated with the relative time of the transition starting from and returning to the other extreme value. The average values of the distance over which the short circuit had to be moved in order to restore conditions to their original state were found to be 4.64 cm and 4.65 cm for the two sets of measurements. Since moving a reflection by half of a wavelength produces no net change in impedance as measured at the generator, the above values for the distance over which the reflection was moved should be helpful in indicating which of the two modes controlled the transition. The wavelength for the π-mode is 9.45 cm, and that for $n = 3$ is 816 cm. The above experimental values correspond more nearly to a half wavelength of the π-mode.

It is therefore apparent that mode competition takes place during this transition, and that conditions in the second mode to oscillate -- the π-mode -- affect the transition more than do conditions in the first. The mode-competition interpretation of this mode change is further supported by the fact that no decay of the 5/3/8 mode preliminary to starting of the π-mode can be observed in the synchroscope trace of the r-f envelope, shown in Fig. 27. In making this observation, it is significant that a broad-band diode detector (100-mc. bandwidth) was used, without a video amplifier between detector and deflecting plates.

c. Mode Stability: Suppression of Unwanted Modes in the 2J39 Magnetron.

An attempt was made to measure mode changes to the type described in the above subsections, using a 2J39 magnetron.[1] This attempt was thwarted by the absence of any modes other than the π-mode. Stable oscillation was observed, for normal magnetron loading, over a range of

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(1) Reference No.1, pp.747-751.

anode voltages extending from somewhere below the theoretical π-mode threshold voltage all the way to d-c cut-off, and for a wide range of values of magnetic field. Thus, none of the usual mode changes or mode stability problems were encountered at all. The range over which π-mode stability was observed is shown in Fig. 28.

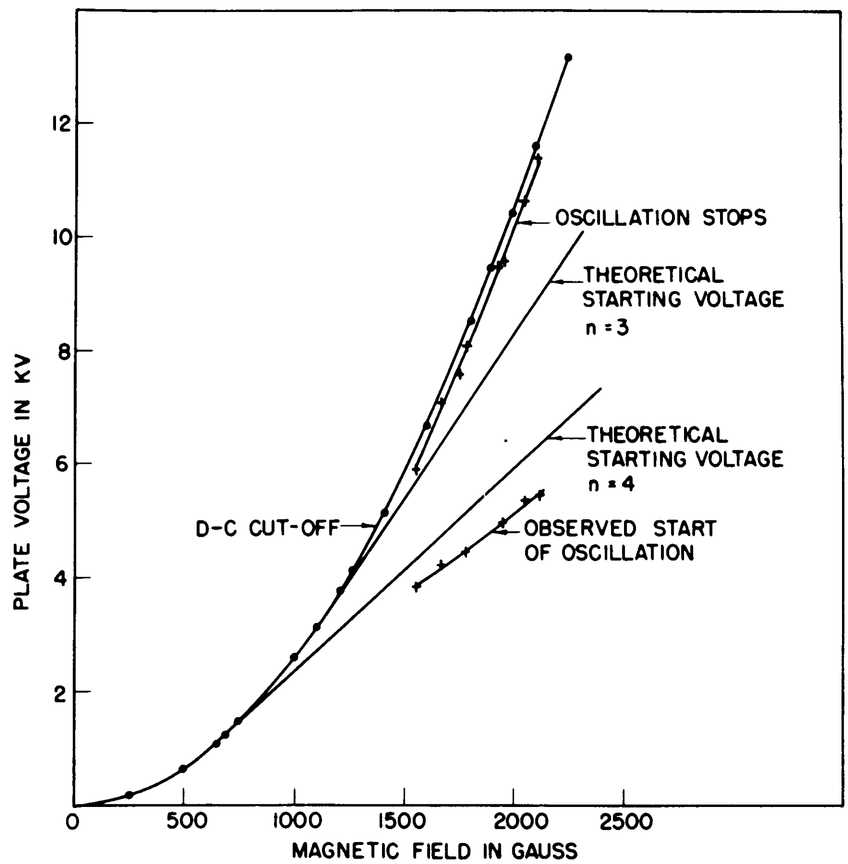

Fig. 28. Limits of π-mode oscillation in 2J39 magnetron.

When the d-c cut-off voltage was approached, and oscillation began to fall off sharply, anode current continued to increase smoothly and with no suggestion of a discontinuity. Theoretical values of anode current which should flow at d-c cut-off were computed[1] for two different values of magnetic field. At 1750 gauss, the theoretical d-c current at cut-off is 21.2 amperes; the maximum current at which stable π-mode oscillation was observed was 19.6 amperes. At 1950 gauss, theoretical d-c current is 25.8 amperes; maximum current for stable π-mode oscillation was 25.2 amperes. Therefore, the theoretical anode current at d-c cut-off was approached very closely before π-mode oscillation ceased.

Another important point shown by Fig. 28 is that stable π-mode oscillation was observed well above the theoretical starting voltage for the 3/3/8 mode. This refutes, if any further refutation is necessary, the mode change theories which have been based primarily on the notion that a lower voltage mode can not exist when the anode voltage reaches the threshold value for the next higher-voltage mode. It is also significant that π-mode operation is observed here at voltage levels in excess of the <u>instability voltage</u> for the 3/3/8 mode.[2] (The <u>instability voltage</u> was taken up in Chapter II.)

The history of the development of the 2J39 magnetron is interesting in the light of the results presented here. Early

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(1) Reference No.5.
(2) Reference No.22; also Reference No.8.

models of the 2J39 experienced π-mode instability at low values of anode current. The problem was studied by Fletcher and Rieke,[1] who recommended changes in the magnet pole pieces which would make the magnetic field in the interaction space more uniform. The model of the 2J39 tested here was one of those in which these changes had been made.

d. Summary

In the preceding subsections, various types of mode change phenomena have been discussed. In all of these, the most fundamental observation expressed by the theoretical work in Chapters IV and V is borne out: that large amplitude oscillation in one mode tends to suppress other modes. In the low-power rising-sun magnetron, it was observed that after oscillation in one mode had been established, other modes of oscillation could not start until the one had collapsed, and each case this collapse was apparently independent of other modes. In the 2J39, suppression of other modes by the π-mode was so effective that the other modes were not observed. Of the magnetrons considered here, only in the 2J54 was the build-up of another mode in competition with the original mode observed. The latter case is by no means in contradiction to the theoretical arguments of Chapters IV and V. In the first place, the maximum amplitude achieved by oscillations in the 5/3/8

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(1) Reference No. 17.

mode, the initially oscillating mode, was very small as compared with π-mode oscillations (see Fig. 27). For reasons explained in Chapter III, it should be expected that the electron bunching the 5/3/8 mode would be less effective than in the π-mode, on account of the presence of a third-order component, as well as the fifth-order component to which the electrons are coupled. Therefore, the suppression of the π-mode by the 5/3/8 mode might well be expected to be relatively ineffective. Furthermore, it was perfectly clear that the presence of the 5/3/8 mode caused an appreciable delay in the starting of the π-mode.

CHAPTER VII

CORRELATION OF THEORY AND EXPERIMENT: SUMMARY AND CONCLUSIONS

1. Mode Competition During Build-Up

The subject of mode competition during the build-up transient has been analyzed rather thoroughly by Rieke.(1) It was stated in Chapter IV that the validity of his analysis depends upon the assumption that when non-linear effects become important, the dependence of the rate of build-up in one mode upon r-f amplitude in the other mode is greater than upon its own amplitude. Rieke stated that this assumption is open to question. Evidently he questioned the assumption only because he has no theoretical justification for it. It is apparent that he realized that it must be true, if mode selection for any one starting transient is to be definite. (This is not to say that different modes cannot be selected on successive pulses, even though the applied pulses are identical.) Otherwise simultaneous stable oscillation in two modes is possible, a condition rarely, if ever, met with in practice.

From a theoretical point of view, Rieke's assumption is supported by the non-linear oscillator theory discussed in Chapter IV. Confirmation of the assumption is most obvious in equations (38) and (39) of that chapter, and these equations are based on the instantaneous voltage-current re-

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(1) Reference No.1, Chapter 8 (by F.F.Rieke).

lationship, $i = ae - be^3$ (cf. equation (2), Chapter IV). It is less obvious, but equally true, that the conditions assumed in Section 5 of Chapter IV, that led to the results shown graphically in Fig. 17, also can not lead to simultaneous stable oscillation in two modes. It is not entirely impossible that some other r-f voltage-current characteristic could lead to circumstances in which two modes of oscillation could be stable simultaneously, but the most plausible types of these relationships do not lead to such results. (Also see Fig. 8.47, Reference No.1, Chapter 8, by F.F.Rieke.) Thus Rieke's assumption, according to him, "open to question," can be supported both theoretically and by observation of actual magnetron performance.

Therefore, magnetron build-up, when conditions allow either of two modes, proceeds, presumably, from random noise, which tends to excite both modes. Build-up in each mode is exponential in form, and more or less independent of the other, until non-linear effects begin to become important. Then competition begins to take place, and the advantage lies more and more with the mode which has greater r-f field intensity for electron bunching, which means especially greater r-f field intensity near the cathode, in the region in which rejection of "unfavorable" electrons takes place. Then it becomes important that the magnitude of oscillation in the mode which has the greater bunching effect should reduce the rate of build-up in the other mode more than the magnitude of oscillation in the other mode should affect its own rate of build-up. This condition causes the mode with more effective bunching to become progressively stronger than the other, and eventually to suppress it altogether. It is occasionally possible for circumstances to be such that different modes are selected on successive pulses.

2. Mode Stability

The mode stability problem can be summarized more quickly than the mode selection problem. The results which have been obtained show that if the anode voltage applied to a magnetron is raised slowly, as compared with the rate of build-up of the mode or modes under consideration, until the starting voltage of another mode has been reached, the advantage lies with the mode which started first, and unless the first mode is relatively weak, the second will not build up. Among all the magnetrons discussed in Chapter VI, there was none in which the π-mode failed as a result of competition with another mode, and only one other mode in one magnetron which failed as a result of competition. In the other cases, any shift which took place from one mode to another was primarily the result of collapse of the first mode, independent of others, rather than the result of competition from the second.

The outstanding example of π-mode stability was the 2J39, in which the upper limit on input voltage, and consequently on input current, was d-c cut-off. From the mode interaction point of view, it is significant to point out that π-mode oscillation continued at a value of anode voltage far above the theoretical threshold voltage of the next higher-voltage mode.

It should be pointed out here that, according to all available information, an increase in magnetic field can be depended upon to increase the upper limit of π-mode stability, in addition to increasing efficiency.

3. Magnetron Design Considerations

The ultimate object of studying the fundamental principles of mode interactions in magnetrons is to improve magnetron design, and methods of magnetron design. A clearer understanding of fundamentals not only makes better magnetrons possible, but makes the process of reaching a good design more direct and less expensive.

The fundamental requirements involving magnetron modes in pulsed magnetrons are, first, the establishment of large-amplitude oscillation in the desired mode (nearly always the π-mode), and second, the maintenance of stable oscillation in that mode for the duration of the applied pulse. In c-w magnetrons, the maintenance of stability is as important as in pulsed magnetrons.

a. R-F Feed-Back

In promoting quick starting, in order to avoid misfiring, or <u>mode skip</u>, and in promoting greater steady-state stability, in order to increase the maximum power obtainable from a magnetron, the r-f feed-back system is very important. It can be altered by changing the loading of the magnetron, or by changing the magnetron structure itself. Of the possible modifications of the structure, the one which has the most effect is changing the ratio of cathode radius to anode radius. The reason for the great effect which this change leads to is the rate at which the intensity of the r-f electric field falls off as the cathode is approached, as discussed in Chapter III. Therefore, the rate of build-up, and to a smaller degree, the steady-state stability, are very sensitive to changes in cathode diameter, with a larger cathode radius leading to faster build-up and more stability. The larger cathode leads incidentally to greater pre-oscillation noise. Since oscillations must build up from noise, more noise means a greater initial amplitude of oscillation, and the length of time required for building up is less. Such an effect has been reported by Forsberg.[1]

(1) Private communication from P.W.Forsberg to F.F.Rieke; reported by Rieke in Chapter 8, p.379, Reference No.1.

Many magnetron designers have improved stability and reduced misfiring by increasing the ratio of cathode radius to anode radius. This was done by the General Electric Co.,Ltd., in England, where it was associated with instability voltages (see Chapter III).[1] It was done by the Bell Telephone Laboratories,[2] who associated with an increase in I_o-- the characteristic current for a given mode in any particular magnetron; this characteristic current was derived from d-c magnetron considerations by Allis, and was applied to the oscillating magnetron by Slater, in absence of any other current which could be clearly defined from theoretical considerations.[3] Finally, it has been done by the Litton Industries, where it was associated with the perveance of a simple diode.[4] However, the sensitivity of misfiring and stability performance of actual magnetrons to this kind of design change is not adequately explained by any of these criteria.

There may be two adverse effects of increasing the cathode radius with respect to the anode radius. The first is the increased stability and the faster build-up of lower-voltage modes, which may delay or prevent the starting of the desired mode. The second is a reduction in electronic efficiency.

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(1) Reference No.22; also Reference No.8.
(2) Reference No. 18.
(3) Reference No. 5.
(4) Reference No. 21.

b. Mode Interactions

The first and best known requirement involving interactions between modes is to keep oscillation in one mode from exciting r-f field components of any other mode. This requires adequate separation of other modes from the desired modes in frequency; the most usual means of separation is the use of strapped anodes or of rising-sun anodes. What constitutes adequate separation has never been clearly set forth. Certainly it must be at least equal to several bandwidths of both the desired mode and of any potentially interfering mode. It may also be desirable to separate modes to the extent that the resonant frequency of a mode whose threshold voltage might be lower than that of the desired π-mode instead becomes higher than that of the π-mode; therefore the unwanted mode has no chance to build up ahead of the π-mode and thus interfere with π-mode starting. For example, in the 2J54 magnetron (see Chapter 6), oscillation in the 5/3/8 mode was found to interfere with the starting of the π-mode; one possible method of removing this interference is to increase the frequency of the $n = 3$ resonance so that the 5/3/8 mode would have a higher starting voltage than the π-mode.

The topic which has been of chief concern here, however, has been the interaction of modes through the electron stream. These results point to the conclusion that once an oscillating mode, for which electron bunching is complete, or almost complete, has been established it is extremely unlikely and perhaps impossible for another mode to build up in its presence, and suppress it. Even when a mode of oscillation is relatively weak, such as the 5/3/8 mode in the 2J54 magnetron, it still can have a considerable adverse effect on a substantially stronger mode. Unless the initially oscillating mode is quite weak, oscillation will shift to another mode primarily as a result of the failure of the initial mode, substantially apart from any effect by the subsequently oscillating mode.

There are two respects in which these conclusions may be applied to magnetron design. The first one is the increase of maximum power in the desired mode. The second is the suppression of unwanted lower voltage modes.

The suppression of unwanted lower voltage modes has been definitely demonstrated to be of importance. A strongly oscillating unwanted mode can prevent the starting of the π-mode until the unwanted mode collapses as a result of its own properties; a less strongly oscillating mode can interfere seriously with π-mode starting, as in the 2J54.

The unwanted modes can be discouraged by systematically making them less stable by interfering with the feed-back mechanism (see sub-section a); it is regrettable that decreasing the cathode diameter for this purpose also affects the π-mode stability adversely. Their r-f field pattern can be distorted, as by strap breaks, which are presumed to affect the π-mode pattern less than that of other modes. However, the effectiveness of strap breaks has never been clearly established. The unwanted modes can also be removed from the region of lower-voltage modes to the region of modes with higher voltages than the π-mode, as described in the above discussion of frequency separation. Now, when the starting voltage of the unwanted mode is slightly higher than that of the π-mode, it can be completely suppressed by a strongly oscillating π-mode; this time, the tendency of the originally oscillating mode to persist against modes which might oscillate under the existing conditions of magnetic field and anode voltage if the original mode were not there works against the unwanted mode.

However, lower-voltage modes are usually not a serious problem. The low-voltage mode in the 2J54 disappeared when a higher magnetic field was used. A similar mode, although theoretically possible in the 2J39 magnetron, was never observed. Furthermore, rising-sun magnetron appears to be inherently free from lower-voltage modes than the π-mode.

Turning our attention now to the application of these principles to extending the maximum power limit of π-mode operation in the face of possible competition from unwanted modes, we find that the presence of other possible modes at the desired operating voltage does not necessarily prevent satisfactory oscillation in the π-mode at this voltage. Instead, we find that it is possible to establish π-mode oscillation in a voltage range suitable for π-mode build-up, after which the voltage may continue upward indefinitely until some inherent instability of the π-mode itself sets in. This kind of a limitation is not well understood from a quantitative point of view, but methods of extending it have been discussed above. It may be recalled here that in a properly loaded 2J39 magnetron, the limitation on π-mode stability coincided with d-c cut-off. Therefore, on the basis of competition from other modes, there appears to be no fundamental limitation on magnetron power. Furthermore, there appears to be no basis yet for predicting the maximum power available from a magnetron from π-mode failure considerations, other than that imposed by d-c cut-off.

c. Magnetic Field Uniformity

A very important factor in stability is the uniformity of the magnetic field. This problem is usually not serious in unpackaged magnetrons, where the magnet is entirely exterior to the magnetron; only a small portion of the magnet gap, both in terms of diameter and in terms of length, is included in the interaction space. This is not the case for packaged magnetrons. In these magnetrons, the magnet gap has been shortened and decreased in diameter by bringing the magnet poles through the end plates and as close to the interaction space as possible. Under these circumstances, it is more difficult to make the magnetic field uniform.

Two examples of improvements in magnetic field uniformity will be discussed here. The 2J39 was changed from a magnetron with unsatisfactory stability characteristics to one with unusually good stability by redesigning the pole pieces to produce a more uniform magnetic field.(1) A relatively uniform magnetic field in the 4J50 magnetron was made possible by means of cathode end shields made of permendur -- a ferromagnetic alloy with a very high Curie temperature.(2)

Reasons for the adverse effects of non-uniform magnetic fields have been discussed in Chapter III. Their importance is emphasized by the great improvement in 2J39 operation by means of small changes in the pole-piece dimensions.

d. Cathode Performance

The need for adequate cathode emission has long been recognized as fundamental to good magnetron oscillation. It has

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(1) Reference No. 17.
(2) Reference No. 3, pp.329-330.

been shown here that good starting and stability characteristics which result from good cathode performance characteristics may be associated with the properties of a magnetron considered as a feed-back amplifier. In a magnetron, the cathode performance is analagous to cathode performance in a triode feed-back oscillator. Good performance in the triode oscillator or in the magnetron is dependent on the loop transmission (defined in Chapter III); the loop transmission may be sub-standard if cathode performance is poor. In the magnetron, it is especially important that the equivalent loop transmission be high for high values of anode voltage, for under these conditions, the tendency for electrons to get out of step with the r-f wave is strong.

4. Suggested Further Research

Obviously, the interaction tests described here were the minimum necessary to demonstrate fundamental principles. Additional, more extensive tests of this type, using widely divergent types of magnetrons, would probably be quite informative. In particular, it might be of interest to find out whether there is any significant change in the interaction phenomena when, with π-mode oscillation taking place, the threshold voltage of the non-oscillating mode under test was reached.

However, there appears to be still more opportunity for further study in considering feed-back phenomena in a magnetron oscillator. This was carried out as far as possible in Chapter III, based primarily on previously existing theory. The primary purpose of discussing it here was to show as well as possible how simple non-linear feed-back oscillator theory is applicable to the magnetron. It seems probable that magnetron operation could be much better understood by further considering magnetrons from this point of view.

One aspect of magnetron design seems to be especially poorly understood. There seems to be room for a substantial amount of experimental work on the subject of the ratio of cathode radius to anode radius, and this topic should also be studied theoretically, based primarily on feed-back concepts. Such a study, if properly carried out, would fill in a considerable gap which now exists in magnetron design procedure.

5. Summary

The principal idea which has come from this research is that large-amplitude oscillation in one mode tends quite strongly to suppress oscillation in other modes. This supports the mode-competition theory advanced by Rieke.(1)

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(1) Reference No.1, Chapter 8 (by F.F.Rieke), pp.380-387.

On the other hand, it contradicts all mode stability and mode change criteria based primarily on the effects of other modes upon a strongly oscillating mode. For example, it is not necessarily true that when the starting voltage for the next higher-voltage mode is reached, the originally oscillating mode will give way to the mode with the higher starting voltage; on the contrary, if first mode is one which oscillates strongly, it is much more likely that the first one will persist, and the second will arise only after conditions are such that the first mode will collapse, primarily independent of conditions in the second mode.

It is therefore true that no fundamental limitation upon high power in magnetrons exists as a result of the possible presence of unwanted modes, provided oscillation in the desired mode can be firmly established. No fundamental limitation on high power in magnetrons could be found which is associated with mode instability, except when d-c cut-off was reached; whether another limit exists or not is still open to question.

Therefore, the fundamental requirement both for quick starting, in order to avoid misfiring, and for stability, with or without the possible presence of other modes, is to establish and maintain effective electron bunching, and this throughout the entire anode length.

APPENDIX I

Numerical Solution of Build-Up Equation

The solution to equation (51) of Chapter IV is to be comsidered here. It will be repeated here, and a new sequence in numbering equations will begin here:

$$d\mathcal{V} = (Rk - 1)\mathcal{V}\,d\tau \qquad k\mathcal{V} < A \qquad (1a)$$

$$d\mathcal{V} = \left(Rk\frac{2\beta + \sin 2\beta}{\pi} - 1\right)\mathcal{V}\,d\tau, \quad k\mathcal{V} > A \qquad (1b)$$

The solution to equation (1a) is obvious. We find that:

$$\ln\mathcal{V} = (Rk - 1)\tau + C \qquad (2)$$

or,
$$\mathcal{V} = \mathcal{V}_0 e^{\sigma\tau} \qquad (3)$$

where $\sigma = (Rk - 1)$ and $\mathcal{V}_0$ is the value of $\mathcal{V}$ when $= 0$.

The solution to (1b) requires a numerical integration; for this purpose it will be rewritten:

$$\Delta\mathcal{V} = \frac{kR\mathcal{V}}{2}\left(\frac{2\beta + \sin 2\beta}{\pi} - \frac{1}{kR}\right)\Delta\tau \qquad (4)$$

Now a sample calculation will be carried out. When reaches the value (A/k), equations (1b) and (4) become significant. At this point $\sin\beta = 1$, $\beta = \pi/2$, and sin $2\beta = 0$, and τ is set equal to zero. As an example, kR will be set equal to 5, and 1/(kR) is then 0.2. To start with, $\Delta\tau$ is arbitrarily selected as 0.01, and equation (4) becomes:

$$\Delta\mathcal{V} = AR(1-0.2)(0.01) \qquad (5)$$
$$= 0.008\,AR$$

The new value of τ at time, $\tau = 0.01$, is therefore $(A/k) + 0.008AR$. Since $kR = 5$, $\mathcal{V}$ now becomes equal to $1.04A/k$; $\sin\beta$ becomes 1/1.04, or 0.9615, β becomes 1.2924, and $\sin 2\beta = 0.5284$. The process which led to (5) may be repeated:

$$\Delta\mathcal{V} = AR\left(\frac{2.5848 - 0.5284}{\pi} - 0.2\right)(0.01) \qquad (6)$$
$$= 0.00791\,AR$$

The process may be continued as far as necessary, and leads to $\mathcal{V}$ as a function of τ as given in Fig. 15. These results are quite consistent with observed magnetron performance.

APPENDIX II

Numerical Solution of Mode Interaction Equation

The equation to be solved here by numerical means is (56) of Chapter IV:

$$F(\mathcal{V}_1, \mathcal{V}_2) = \frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{2\pi} \psi(\mathcal{V}_1 \cos \omega_1 t + \mathcal{V}_2 \cos \omega_2 t) \cos \omega_1 t \, d(\omega_1 t)\, d(\omega_2 t) \quad (7)$$

The expression, $\psi(v)$, is defined as (a tanh bv), as expressed in equation (58) of Chapter IV. In the above double integration, $\omega_1 t$ and $\omega_2 t$ are to be considered as independent variables. For integrating with respect to $\omega_1 t$, $\mathcal{V}_2 \cos \omega_2 t$ may be replaced by c, and a new integral written:

$$G_1(\mathcal{V}_1, c) = \frac{1}{\pi}\int_0^{2\pi} \psi(\mathcal{V}_1 \cos \omega_1 t + c) \cos \omega_1 t \, d(\omega_1 t) \quad (8)$$

Since the integrand takes on the same set of values between 0 and π as between π and 2π, with only the order reversed, numerical integration will be carried out only between 0 and π. Therefore, (8) may be replaced by:

$$G_1(\mathcal{V}_1, c) = \frac{2}{\pi}\int_0^{\pi} \psi(\mathcal{V}_1 \cos \omega_1 t + c) \cos \omega_1 t \, d(\omega_1 t) \quad (9)$$

The range between 0 and π will be divided into 16 parts, and integrated according to Simpson's rule.[1] A sample calculation will be carried out for $\mathcal{V}_1 = 2$, $c = 0.5$:

(1) $\omega_1 t$	(2) $\cos \omega_1 t$	(3) $2 \cos \omega_1 t$	(4) $(2 \cos \omega_1 t + 0.5)$	(5) $\cos \omega_1 t \tanh x$	(6) Multiplying Factor for Simpson's Rule
0	1.000	2.000	2.5	0.987	1
$\pi/16$	0.9808	1.962	2.462	0.967	4
$\pi/8$	0.9239	1.848	2.348	0.907	2
$3\pi/16$	0.8315	1.663	2.163	0.810	4
$\pi/4$	0.7071	1.414	1.914	0.678	2
$5\pi/16$	0.5556	1.111	1.611	0.513	4
$3\pi/8$	0.3827	0.765	1.256	0.327	2
$7\pi/16$	0.1951	0.390	0.890	0.139	4
$\pi/2$	0	0	0.500	0	2
$9\pi/16$	-0.1951	-0.390	0.110	-0.021	4
$5\pi/8$	-0.3827	-0.765	-0.265	+0.099	2
$11\pi/16$	-0.5556	-1.111	-0.611	0.303	4
$3\pi/4$	-0.7071	-1.414	-0.914	0.512	2
$13\pi/16$	-0.8315	-1.663	-1.163	0.685	4
$7\pi/8$	-0.9239	-1.848	-1.348	0.808	2
$15\pi/16$	-0.9808	-1.962	-1.462	0.882	4
π	-1.000	-2.000	-1.500	0.906	1

When each quantity in column (5) is multiplied by the

(1) See Burington, "<u>Mathematical Tables and Formulas</u> (Handbook Publishers, Inc., Sandusky, O., Second Edition, 1946), p.13.

appropriate factor in column (7) and the results are added, the sum is 25.667. This sum is multiplied by h/3, where h is the interval between one value of $\omega_1 t$ and the next -- $\pi/16$ in this case.

$$G_1(2, 0.5) = \frac{2}{\pi} \cdot \frac{\pi}{16} \cdot \frac{1}{3}(25.667)$$

$$= 1.069 \tag{10}$$

Values of G_1 as a function of c, for $\mathcal{V}_2 = 0.5$, 1.0, and 2.0, are shown in Fig. 29. It is important to point out that $G_1(\mathcal{V}_1, c)$ is equal to $G_1(\mathcal{V}_1, -c)$.

In order to find $F_1(\mathcal{V}_1, \mathcal{V}_2)$ it is necessary to carry out a further integration:

$$F_1(\mathcal{V}_1, \mathcal{V}_2) = \frac{1}{2\pi} \int_0^{2\pi} G_1(\mathcal{V}_1, \mathcal{V}_2 \cos \omega_2 t)\, d(\omega_2 t) \tag{11}$$

Another numerical integration of the same kind as described above will be carried out. Values of G_1 for each value of $\mathcal{V}_2 \cos \omega_2 t$ can be found from Fig. 29. As a result of the fact that $G_1(\mathcal{V}_1, c)$ is equal to $G_1(\mathcal{V}_1, -c)$, equation (11) becomes:

$$F_1(\mathcal{V}_1, \mathcal{V}_2) = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} G_1(\mathcal{V}_1, \mathcal{V}_2 \cos \omega_2 t)\, d(\omega_2 t) \tag{12}$$

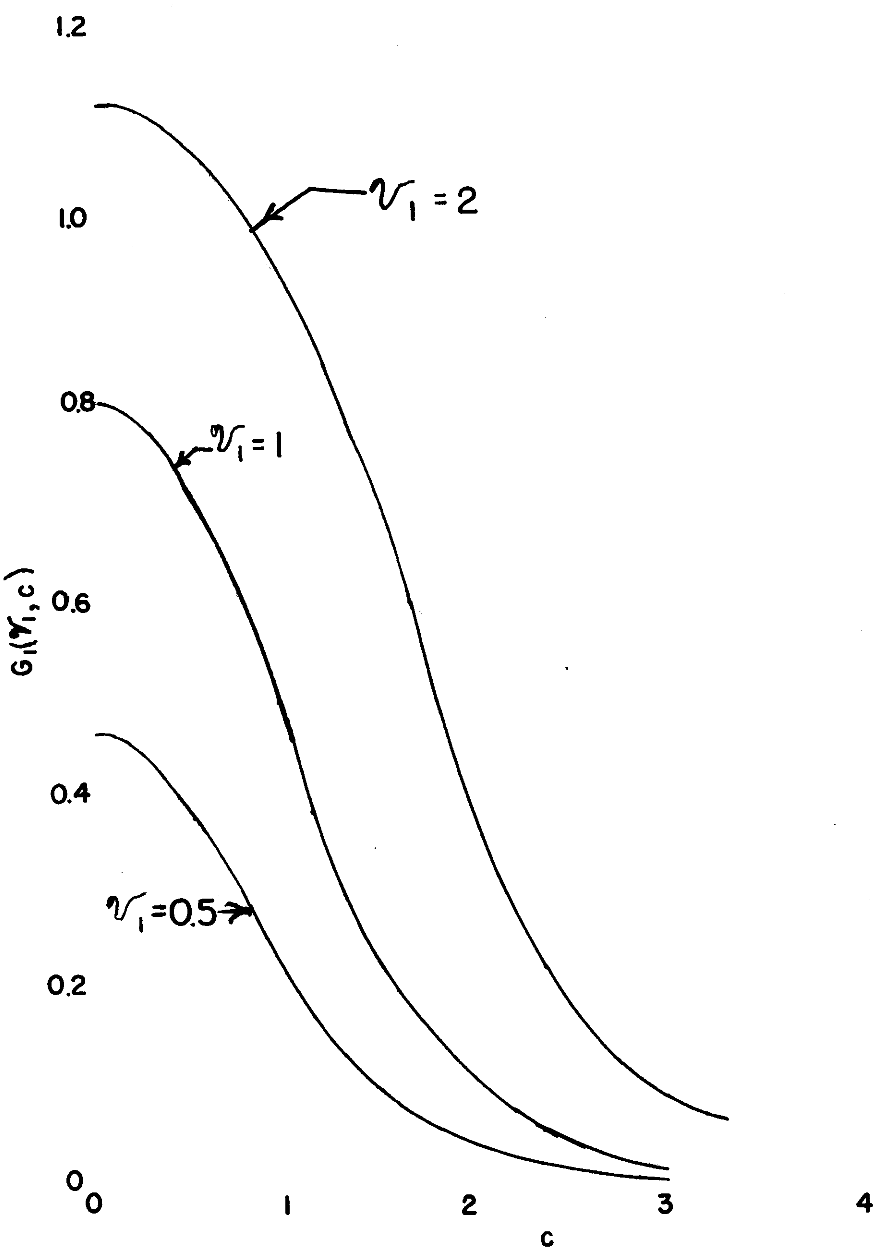

Figure 29. Values of $G_1(\mathcal{V}_1, c)$--see text.

In carrying out a sample numerical integration for (12), $\mathcal{V}_1 = 2$ and $\mathcal{V}_2 = 0.5$ will be used:

(1) $\omega_2 t$	(2) 0.5 cos $\omega_2 t$	(3) G(2, 0.5 cos $\omega_2 t$)	(4) Multiplying Factor for Simpson's Rule
0	0.5	1.069	1
$\pi/16$	0.4904	1.072	4
$\pi/8$	0.4620	1.076	2
$3\pi/16$	0.4158	1.082	4
$\pi/4$	0.3536	1.092	2
$5\pi/16$	0.2778	1.102	4
$3\pi/8$	0.1914	1.111	2
$7\pi/16$	0.0926	1.116	4
$\pi/2$	0	1.119	1

After each term in column (4) has been multiplied by the appropriate factor, the sum is 26.234. Therefore,

$$F_1(2, 0.5) = \frac{2}{\pi} \cdot \frac{1}{3} \cdot \frac{\pi}{16}(26.234) = 1.09 \qquad (13)$$

In the expression, $\psi(v) = a \tanh bv$, both a and b were tacitly assumed to be equal to one when $\psi(v)$ was substituted into the integral by which $F_1(\mathcal{V}_1, \mathcal{V}_2)$ was expressed. This condition can be easily remedied by substituting $b\mathcal{V}_1$ and $b\mathcal{V}_2$ for $\mathcal{V}_1$ and $\mathcal{V}_2$, respectively, and multiplying each value for F_1 by a. For example, (13) becomes:

$$F_1(b\mathcal{V}_1, b\mathcal{V}_2) = F_1(2.0, 0.5) = 1.09a \tag{14}$$

The results obtained by this means are approximate, and somewhat incomplete. However, they are adequate to demonstrate the general character of the stability conditions.

APPENDIX III

Equipment Used in Mode Interaction Experiment

Synchroscope: M. I. T. Radiation Laboratory Model P4E. This supplies a synchronizing signal to the Model 12 Pulser (see below) and to the low-power pulser; it is also used for monitoring synchronization between the outputs of the two pulsers, and, in conjunction with a capacity voltage divider, for measuring the anode voltage on the 718EY magnetron.

High-Power Pulser: Radiation Laboratory Model 12; 250 kw. maximum power; 50-ohm 5-microsecond pulse-forming network used, switched by hydrogen thyratron.

Low-Power Pulser: Designed and constructed especially for this experiment; M. I. T. Research Laboratory of Electronics drawing no. A-816-7; variable pulse duration; 2 kv. maximum pulse output. All of the essential features of this unit were copied after the driver unit of the Radiation Laboratory Model 9 Pulser (see Glasoe, Lebacqz, Pulse Generators, McGraw-Hill, 1948, pp. 157-159), as modified for 0.5 to 5.0 microsecond continuously variable pulse duration. (The second 3E29 is used as the switch tube for the magnetron.) A separate power supply, with d-c outputs of 2.0 kv. and 0-500 volts, was used (C. W. Magnetron Power Supply, Model F-5000, M. I. T. Research Laboratory of Electronics). The low-power pulser supplies anode voltage for both the QK-61 magnetron and the klystron amplifier.

Signal Source for Reference Frequency: TVN-7BL power supply (Browning Laboratories, Winchester, Mass.) with 2K41 klystron.

APPENDIX IV

Construction of Low-Power Rising-Sun Magnetron

The essential dimensions of this magnetron are:

Cathode radius: 0.0625 in.

Anode radius: 0.101 in.

Anode height: 0.388 in.

Radius to bottom of small cavities (from axis of cathode): 0.678 in.

Radius to bottom of large cavities: 1.017 in.

The output coupling was of the conventional loop type, and the loop was attached to a seven-eighths inch coaxial line. By measurement, the following values for loading of the π-mode were found:

External Q: 780

Unloaded Q: 1327

Loaded Q (for matched transmission line): 490

The cathode was oxide-coated, with no screen or nickel matrix, on a "grade A" nickel sleeve, and mounted radially. Diameter of the end shields was 0.180 inch.

ABSTRACT

It has been the object of this research to investigate the problem of establishing and maintaining the desired mode of operation in multi-cavity magnetrons, and particularly, the cause of the shifting of oscillation from one mode to another. The fundamental question was whether such a shift depends primarily upon the originally oscillating mode, upon the second mode, or upon competition between the two. The preponderance of evidence points to the failure of the initial mode as the fundamental cause in most cases, especially if the initial mode is very strongly oscillating before the mode change.

The history of mode problems is briefly reviewed. It is shown how the operation of magnetrons was quite drastically improved by separation of the resonant frequencies of other modes from that of the desired mode, first by means of "mode locking" straps, and second by means of the "rising-sun" anode structure. It is also shown how it was easy for those working with magnetrons to draw inaccurate conclusions concerning the fundamental causes of mode shifts. A very important contribution to the understanding of mode problems was made by Rieke, who made clear the distinction between failure to start in the desired mode (misfiring), and a shift from one mode

to another after oscillation had started in the first mode.

The magnetron is considered in terms of its properties as a feed-back oscillator in order to supply background for the non-linear circuit which follows. The bunching mechanism in a magnetron is shown to have the same kind of properties as feed-back in oscillators in which the feed-back is associated with a clearly distinguishable part of the circuit. Bunching in a magnetron is broken down into two parts: rejection of electrons which are in such phase as to take energy from the r-f system, and phase-focusing, which maintains electrons in such phase as to give up energy to the r-f system.

The non-linear oscillator theory developed by van der Pol is studied, and it is shown how this kind of analysis is applicable to magnetrons as well as to triode oscillators. An approximate solution for the build-up equation for one mode is given, and the interaction problem when there are two resonances in the passive circuit is studied. Results for the two-mode problem show not only that the presence of large-amplitude oscillations corresponding to one mode of oscillation tends to suppress oscillation in the other mode, but that the effect of the amplitude of oscillations in one mode upon its own rate of build-up is less than the effect of the amplitude of oscillations in the other mode upon the rate of build-up in the first mode. It is this latter conclusion which Rieke stated as an assumption in his analysis of mode interactions during build-up.

Certain shortcomings in van der Pol's assumption as to the character of the non-linearity in the feed-back oscillators which he has discussed are pointed out, and other functions are suggested to represent the relationship between instantaneous voltage and current. It is shown that the linearization of the problem in an oscillator with a high-Q resonant circuit consists of deriving a relationship between the magnitudes of sinusoidal voltages and currents which results from expressing current as a non-linear function of a sinusoidal voltage; only the fundamental component of the resulting Fourier series is considered. Again the build-up problem for one mode is considered, and it is shown that this theoretical build-up transient is more nearly in agreement with observed magnetron build-up than the one derived from van der Pol's assumptions. The interaction between modes in a two-mode system is also considered, and the results are found to be similar in principle to those derived from using van der Pol's assumptions.

An approximate analysis, of electron motion in an oscillating plane magnetron is carried out. A large-amplitude r-f travelling wave is assumed, and the effect of a small-amplitude perturbation is considered. It is found that the perturbed electron motion is such that the electron absorbs energy from the perturbing wave. This agrees with the conclusion drawn from non-linear oscillator theory that a strongly oscillating mode tends to suppress other modes.

Some observations of magnetron performance are presented which support the above theoretical principles. The value of Q for one resonant mode was measured while oscillation in another mode was taking place, and it was found that the Q of the non-oscillating mode was quite markedly lowered by the presence of oscillations in another mode. The above theoretical principles are indirectly supported by observations of mode changes; in three cases it was found that mode changes depended primarily on the initially oscillating mode, and in a fourth case the starting of a strongly oscillating mode was appreciably delayed by competition from a weakly oscillating mode. In still another magnetron, no mode other than the one for which the magnetron was designed was found to oscillate, even though other modes were theoretically possible; the only limitation on anode current and voltage, with stable oscillation taking place, was found at d-c cut-off, where anode current could flow without the presence of oscillation. The latter observation was found to be true for several values of magnetic field in that magnetron.

It is therefore concluded that if effective electron bunching can be maintained, the desired mode in a magnetron does not need to be limited in power by competition from other modes. It is pointed out that effective bunching throughout the axial length of the magnetron depends upon magnetic field uniformity. It is also concluded that any mode-change criterion which depends primarily upon conditions in a second mode which might build up and suppress the first is unsatisfactory.

No fundamental reason has been found which places an upper limit on maximum power output for which the desired mode of oscillation in a magnetron is stable, provided that it is possible to increase the feed-back intensity, although it may be possible that such a limit exists.

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BIOGRAPHICAL NOTE

The author was born in Jacksonville, Illinois, on February 14, 1921. He attended the Indianola, Iowa, public schools and graduated from Indianola High School in May, 1938.

He attended Simpson College from September, 1938, to May, 1940. He was designated for Annual Honors for both academic years, and was elected to Pi Kappa Delta (forensic fraternity).

He entered Iowa State College in September, 1940, and received the degree of Bachelor of Science in Electrical Engineering on March 19, 1943. He was elected to membership in Pi Mu Epsilon (mathematical fraternity) and Phi Kappa Phi (scholarship honorary fraternity).

He served on active duty in the United States Navy from May, 1943, to February, 1946. He attended the Naval Reserve Midshipmen's School, Annapolis, Maryland, from May to August, 1943, and was commissioned Ensign, U. S. Naval Reserve. From September, 1943, to July, 1944, he was Instructor in Electrical Engineering at the U. S. Naval Academy. Following this, he attended the Naval Training School (Pre-Radar) at Harvard University, and the Naval Training School (Radar) at M. I. T., completing this work in December, 1944. From January, 1945, to February, 1946, he was Radar Field Engineer in the Electronic Field Service Group, Washington, D.C.

From February to May, 1946, he was Instructor in Mathematics at Simpson College, filling a vacancy caused by the death of a professor.

In June, 1946, he was appointed Research Assistant in Electrical Engineering at M. I. T. and was assigned to the High-Power Magnetron Group at the Research Laboratory of Electronics. This assignment has continued from that time to the present. He was promoted to Research Associate in July, 1948. He received the degree of Master of Science in Electrical Engineering on September 26, 1947. The thesis presented in connection with this degree was entitled "Mode Stability in Resonant-Cavity Magnetrons." He is also the author of Research Laboratory of Electronics Technical Report No. 99, entitled "Investigation of Anode Structure in Rising-Sun Magnetrons." While at M. I. T. he has been elected an associate member of Sigma Xi.